8.12 Washer Method: Revolve Around Other Axes
Calculus

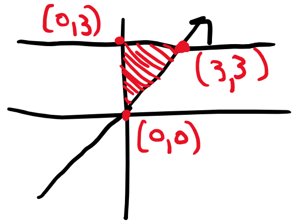
Solutions

Practice

1. A region S is bounded by the graphs of $y = x$, $x = 0$, and $y = 3$.

$x = y$

a. Sketch the graph and find the area of region S.



$$A = \int_0^3 (3 - x)\,dx = 4.5$$

b. Let S be the base of a solid with cross sections perpendicular to the x-axis that form a square. Find the volume of this solid. [Use a calculator after you set up the integral.]

$$V = \int_0^3 (3-x)^2\,dx$$

$$V = 9$$

c. Let S be the base of a solid with cross sections perpendicular to the y-axis that form a semi-circle. Find the volume of this solid. [Use a calculator after you set up the integral.]

$$V = \int_0^3 \frac{\pi}{2}\left(\frac{y}{2}\right)^2 dy$$

$$V = 3.534$$

Write the equation for the "big radius" and the "little radius" for the solid of revolution when revolving S around the given line. Then setup the integral to find the volume of the solid formed. **DO NOT EVALUATE.**

d. The line $x = 3$.

$R = 3$

$r = 3 - y$

$V = \pi \int_0^3 \left[9 - (3-y)^2\right] dy$

e. The line $y = -1$.

$R = 4$

$r = x + 1$

$V = \pi \int_0^3 \left[16 - (x+1)^2\right] dx$

f. The line $x = -1$.

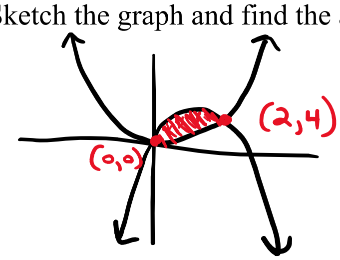
$R = y + 1$

$r = 1$

$V = \pi \int_0^3 \left[(y+1)^2 - 1\right] dy$

2. A region T is bounded by the graphs of $y = x^2$ and $y = 4x - x^2$.

a. Sketch the graph and find the area of region T.



$x^2 = 4x - x^2$

$2x^2 - 4x = 0$

$2x(x - 2) = 0$

$x = 0 \quad x = 2$

$$A = \int_0^2 (4x - x^2) - x^2\,dx$$

$$A = \int_0^2 (4x - 2x^2)\,dx$$

$$A = 2.667$$

b. Let T be the base of a solid with cross sections perpendicular to the x-axis that form a semicircle. Find the volume of this solid. [Use a calculator after you set up the integral.]

$$V = \int_0^2 \frac{\pi}{2}\left(\frac{4x - 2x^2}{2}\right)^2 dx$$

$$V \approx 1.6755$$

Write the equation for the "big radius" and the "little radius" for the solid of revolution when revolving S around the given line. Then setup the integral to find the volume of the solid formed. **DO NOT EVALUATE.**

c. The line $y = 6$.

$R = 6 - x^2$

$r = 6 - (4x - x^2) = x^2 - 4x + 6$

$V = \pi \int_0^2 \left[(6-x^2)^2 - (x^2-4x+6)^2\right] dx$

d. The line $y = -3$.

$R = 3 + 4x - x^2$

$r = 3 + x^2$

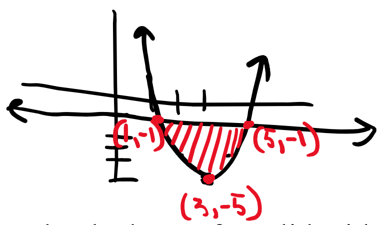
$V = \pi \int_0^2 \left[(3+4x-x^2)^2 - (3+x^2)^2\right] dx$

3. A region D is bounded by the graphs of $y = (x-3)^2 - 5$ and $y = -1$.

a. Sketch the graph and find the area of region D.

$\pm\sqrt{y+5} = x - 3$

$x = 3 \pm \sqrt{y+5}$



$y = x^2 - 6x + 9 - 5$

$y = x^2 - 6x + 4$

$A = \int_1^5 \left[(-1) - (x^2 - 6x + 4)\right] dx$

$A = \int_1^5 (-x^2 + 6x - 5)\, dx$

$A = 10.667$

b. Let D be the base of a solid with cross sections perpendicular to the x-axis that form a rectangle with a height 3 times the width. Find the volume of this solid. [Use a calculator after you set up the integral.]

$V = \int_1^5 \left[3(-x^2 + 6x - 5)^2\right] dx$

$V = 102.4$

c. Let D be the base of a solid with cross sections perpendicular to the y-axis that form an isosceles right triangle. Find the volume of this solid. [Use a calculator after you set up the integral.]

$V = \int_{-5}^{-1} \frac{1}{2}\left[(3+\sqrt{y+5}) - (3-\sqrt{y+5})\right]^2 dy$

$V = \int_{-5}^{-1} \frac{1}{2}(2\sqrt{y+5})^2\, dy$

$V = 16$

Write the equation for the "big radius" and the "little radius" for the solid of revolution when revolving D around the given line. Then setup the integral to find the volume of the solid formed. **DO NOT EVALUATE.**

d. The line $y = 9$.

$R = x^2 - 6x + 4 - 9$

$r = 10$

$V = \pi \int_1^5 \left[(x^2 - 6x - 5)^2 - (100)\right] dx$

e. The line $x = -2$.

$R = 3 + \sqrt{y+5} + 2$

$r = 3 - \sqrt{y+5} + 2$

$V = \pi \int_{-5}^{-1} \left[(5+\sqrt{y+5})^2 - (5-\sqrt{y+5})^2\right] dy$

f. The line $x = 5$.

$R = 5 - (3 - \sqrt{y+5})$

$r = 5 - (3 + \sqrt{y+5})$

$V = \pi \int_{-5}^{-1} \left[(2+\sqrt{y+5})^2 - (2-\sqrt{y+5})^2\right] dy$

No test prep for this lesson.