Calculus

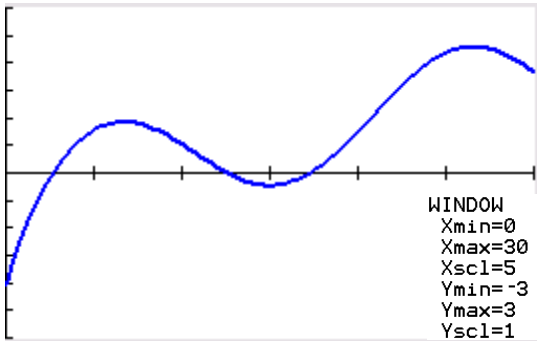
Write your questions and thoughts here!

8.2 Connecting Pos, Vel, Acc with Integrals **Notes**

1. A particle moves along the x-axis with an acceleration of $a(t) = 12t - 4$. The particle's velocity is 18 centimeters per second at $t = 2$. The initial position of the particle is 8 cm. What is a function, $x(t)$ that represents the position of the particle?

2. Mr. Brust is driving across town to Mr. Sullivan's house to play with a new set of Star Wars figures. Mr. Brust's speed would obviously vary throughout the drive, but because he is so cool, he came up with a function that represents his velocity (miles per minute) at any time t (minutes) since he left his house during the 30 minute drive.

   $v(t) = \sin(0.3t) + \ln(t + 1) - 2$

   Set up the expressions for the following scenarios. Use a calculator to solve.

   a. How far is Mr. Brust from his house after 10 minutes?

   b. How far is Mr. Brust from his house after 15 minutes?

   c. If Mr. Brust arrives at Mr. Sullivan's house after 30 minutes, how far away does he live?

   d. How many miles did Mr. Brust drive?

$\int$ velocity = $\int$ |velocity| =

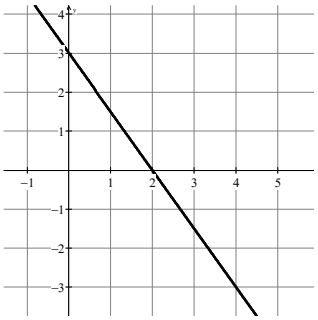
Don't get this confused with: |velocity| =

Write your questions and thoughts here!

3. A particle's velocity is given by $v(t) = 4t^3 - 6t^2 + 1$. The function $x(t)$ represents the position of the particle along the x-axis.
   a. Find the position of the particle after 3 seconds if $x(0) = 5$.

   b. Find the position of the particle after 2 seconds if $x(1) = -2$.

4. What is the total distance traveled by a particle during the first 4 seconds if the particle's velocity function is given by $v(t) = -1.5t + 3$? Show the set up AND your work.



8.2 Connecting Pos, Vel, Acc with Integrals

Practice

Calculus

1. A coin is dropped from an 850-ft building. The velocity of the coin is $v(t) = -32t$ feet per second. Find both the position function and acceleration function.

2. A particle moves along the y-axis with an acceleration of $a(t) = 2$ where t is time in seconds. The particle's velocity at $t = 2$ is 5 cm/sec. The position of the function at $t = 2$ is 10 cm. What is the position of the particle at $t = 6$?

3. A ball is thrown down off of a house with a velocity of $v(t) = -32t - 8$ where t is time in seconds and v is ft/sec. The ball is 20 feet in the air at $t = 1$. What is the initial height of the ball?

4. A particle moves along the y-axis with an acceleration of $a(t) = 12t - 6$ with initial velocity of -10 and initial position 0. Find the position of the function at the particle's minimum velocity.

5. **Calculator active**. A particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \frac{2}{t^2+3}$. If the position of the particle is $x = 2$ when $t = 4$, what is the position of the particle when $t = 6$?

6. **Calculator active**. An object moves along the y-axis with initial position $y(0) = 1$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \cos(\pi t)$. What is the position of the object at time $t = 3$?

7. Mr. Kelly leaves for a trip at 3:00 p.m. (time $t = 0$) and drives with velocity $v(t) = 60 - \frac{1}{2}t$ miles per hour, where t is measured in hours.
   a. Find $\int_0^2 v(t)\,dt$

   b. Explain the meaning of your answer to part *a* in the context of this problem.

8. A particle's velocity is given by $v(t) = 2t - 8$, where t is measured in seconds, v is measured in feet per second, and $s(t)$ represents the particle's position.
   a. If $s(0) = 2$, what is the value of $s(3)$?

   b. What is the net change in distance over the first 5 seconds?

   c. What is the total distance traveled by the particle during the first 5 seconds? Show the set up AND your work.

9. A particle's velocity is given by $v(t) = t^2 + 2t - 15$, where t is measured in minutes, v is measured in meters per minute, and $s(t)$ represents the particle's position.
   a. If $s(1) = -3$, what is the value of $s(3)$?

   b. What is the net change in distance over the first 5 minutes?

   c. What is the total distance traveled by the particle during the first 5 minutes? Show the set up AND your work.

10. **Calculator active.** A particle's velocity is given by $v(t) = 6\cos 3t$, where t is measured in days, v is measured in yards per day, and $s(t)$ represents the particle's position.
    a. If $s(0) = 5$, what is the value of $s\left(\frac{\pi}{2}\right)$? Calculator allowed.

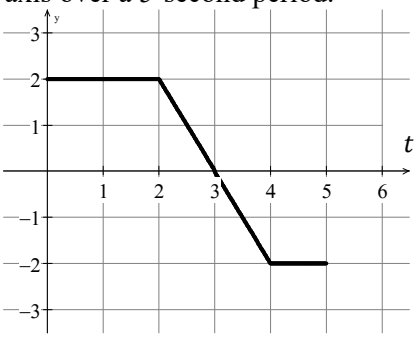
    b. What is the net change in distance over the first $\frac{\pi}{2}$ days? Calculator allowed.

    c. What is the total distance traveled by the particle during the first $\frac{\pi}{2}$ days? Show the set up and use a calculator to find the answer.

11. The graph to the right shows the velocity of an object moving along the x-axis over a 5-second period.
    a. If the objected started 2 meters to the right, where is the object after 3 seconds?

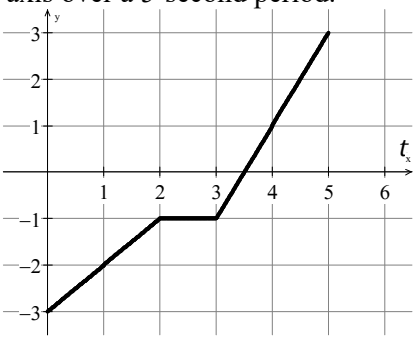
    b. Where is the object after 5 seconds?

    c. Find the total distance traveled by the object over the 5-second period.



12. The graph to the right shows the **velocity** of an object moving along the x-axis over a 5-second period.
    a. Find the total distance traveled by the object over the 5-second period.

    b. At time $t = 2$, the particle is at the point where $x = 10$. Where was the particle at time $t = 0$?



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Test Prep

13. **Calculator active.** At time t, $0 < t < 2.5$, the velocity of a particle moving along the x-axis is given by $v(t) = t\cos(t^2)$. Let $t = b$ be the time at which the particle changes direction from moving left to moving right. What is the total distance traveled by the particle during the time $0 < t < b$?

(A) 0.5 (B) 1.253 (C) 1.5 (D) 2.171

This next problem is a common type of problem on an AP exam. Make sure you understand it!

14. **Calculator active.** Mr. Kelly and Mr. Sullivan are doing a morning speed-walk race going down a straight street. For $0 \le t \le 20$, Mr. Kelly's velocity at time t is given by $K(t) = \frac{16500}{t^2-5t+74.33}$ and Mr. Sullivan's velocity at time t is given by $S(t) = 41t^3e^{-0.6t}$. Both $K(t)$ and $S(t)$ are positive for $0 \le t \le 20$ and are measured in yards per minute, and t is measured in minutes. Mr. Kelly has a 5 yard head-start at $t = 0$, and is ahead of Mr. Sullivan for the entire time $0 \le t \le 20$.

a. Find the value of $\frac{1}{5}\int_{10}^{15} K(t)\,dt$. Using correct units, interpret the meaning of $\frac{1}{5}\int_{10}^{15} K(t)\,dt$ in the context of the problem.

b. At time $t = 7$, is Mr. Kelly speeding up or slowing down? Give a reason for your answer.

c. Is the distance between Mr. Kelly and Mr. Sullivan at time $t = 7$ increasing or decreasing? Give a reason for your answer.

d. What is the maximum distance between Mr. Kelly and Mr. Sullivan over the time interval $0 \le t \le 20$? Justify your answer.