8.2 Connecting Pos, Vel, Acc with Integrals
Calculus

Solutions

Practice

1. A coin is dropped from an 850-ft building. The velocity of the coin is $v(t) = -32t$ feet per second. Find both the position function and acceleration function.

$a(t) = -32$ $h(t) = -16t^2 + 850$

2. A particle moves along the y-axis with an acceleration of $a(t) = 2$ where t is time in seconds. The particle's velocity at $t = 2$ is 5 cm/sec. The position of the function at $t = 2$ is 10 cm. What is the position of the particle at $t = 6$?

$V(t) = \int 2\,dt$
$V = 2t + C$
$5 = 4 + C$
$1 = C$

$x(t) = \int (2t+1)\,dt$
$x(t) = t^2 + t + C$
$10 = 4 + 2 + C$
$4 = C$

$x(t) = t^2 + t + 4$
$36 + 6 + 4$
$\boxed{x(6) = 46 \text{ cm}}$

3. A ball is thrown down off of a house with a velocity of $v(t) = -32t - 8$ where t is time in seconds and v is ft/sec. The ball is 20 feet in the air at $t = 1$. What is the initial height of the ball?

$h(t) = \int (-32t - 8)\,dt$
$h(t) = -16t^2 - 8t + C$
$20 = -16 - 8 + C$ $C = 44$

$h(t) = -16t^2 - 8t + 44$

$\boxed{44 \text{ ft.}}$

4. A particle moves along the y-axis with an acceleration of $a(t) = 12t - 6$ with initial velocity of -10 and initial position 0. Find the position of the function at the particle's minimum velocity.

Min Velocity when $v' = 0$

$12t - 6 = 0$

$t = \frac{1}{2}$

$v(t) = 6t^2 - 6t + C$

$v(t) = 6t^2 - 6t - 10$

$y(t) = 2t^3 - 3t^2 - 10t$

$y(\frac{1}{2}) = 2(\frac{1}{8}) - 3(\frac{1}{4}) - 10(\frac{1}{2})$

$= \frac{1}{4} - \frac{3}{4} - 5$

$\boxed{-5.5}$

5. **Calculator active**. A particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \frac{2}{t^2+3}$. If the position of the particle is $x = 2$ when $t = 4$, what is the position of the particle when $t = 6$?

$x(6) = 2 + \int_4^6 v(t)\,dt = \boxed{2.147}$

6. **Calculator active**. An object moves along the y-axis with initial position $y(0) = 1$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \cos(\pi t)$. What is the position of the object at time $t = 3$?

$y(3) = 1 + \int_0^3 v(t)\,dt = \boxed{1}$

7. Mr. Kelly leaves for a trip at 3:00 p.m. (time $t = 0$) and drives with velocity $v(t) = 60 - \frac{1}{2}t$ miles per hour, where t is measured in hours.

a. Find $\int_0^2 v(t)\,dt$

$60t - \frac{1}{4}t^2 \Big|_0^2$

$[60\cdot(2) - \frac{1}{4}(4)] - [0]$

$120 - 1 = \boxed{119}$

b. Explain the meaning of your answer to part *a* in the context of this problem.

2 hours into his trip, Mr. Kelly is 119 miles from his starting position

8. A particle's velocity is given by $v(t) = 2t - 8$, where t is measured in seconds, v is measured in feet per second, and $s(t)$ represents the particle's position.

a. If $s(0) = 2$, what is the value of $s(3)$?

$$S(3) = 2 + \int_0^3 (2t-8)\,dt$$
$$2 + \left[t^2 - 8t\right]\Big|_0^3$$
$$2 + \left[(9-24) - (0)\right] \rightarrow 2 + (-15)$$
$$\boxed{S(3) = -13}$$

b. What is the net change in distance over the first 5 seconds?

$$\int_0^5 (2t-8)\,dt$$
$$t^2 - 8t \Big|_0^5$$
$$(25-40) - (0) = \boxed{-15 \text{ feet}}$$

c. What is the total distance traveled by the particle during the first 5 seconds? Show the set up AND your work.

$$2t - 8 = 0$$
$$2t = 8$$
$$t = 4$$

$$\left|\int_0^4 (2t-8)\,dt\right| + \left|\int_4^5 (2t-8)\,dt\right|$$
$$\left|\left[t^2 - 8t\right]\right|\Big|_0^4 + \left|\left[t^2 - 8t\right]\right|\Big|_4^5$$
$$\left|(16-32) - 0\right| + \left|(25-40) - (16-32)\right|$$
$$|-16| + |-15 - -16| = \boxed{17 \text{ feet}}$$

9. A particle's velocity is given by $v(t) = t^2 + 2t - 15$, where t is measured in minutes, v is measured in meters per minute, and $s(t)$ represents the particle's position.

a. If $s(1) = -3$, what is the value of $s(3)$?

$$S(3) = -3 + \int_1^3 (t^2 + 2t - 15)\,dt$$
$$-3 + \left[\tfrac{1}{3}t^3 + t^2 - 15t\right]\Big|_1^3$$
$$-3 + \left[9 + 9 - 45\right] - \left[\tfrac{1}{3} + 1 - 15\right]$$
$$-3 + (-27) - \left(\tfrac{1}{3} - 14\right) = -\tfrac{49}{3} \approx \boxed{-16.333 \text{ meters}}$$

b. What is the net change in distance over the first 5 minutes?

$$\int_0^5 (t^2 + 2t - 15)\,dt$$
$$\left[\tfrac{1}{3}t^3 + t^2 - 15t\right]\Big|_0^5$$
$$\left[\tfrac{1}{3}(125) + 25 - 75\right] - 0 = \tfrac{125}{3} - \tfrac{150}{3} = \boxed{-\tfrac{25}{3} \text{ meters}}$$

c. What is the total distance traveled by the particle during the first 5 minutes? Show the set up AND your work.

$$t^2 + 2t - 15 = 0$$
$$(t+5)(t-3) = 0$$
$$t = -5 \quad t = 3$$

$$\left|\int_0^3 (t^2 + 2t - 15)\,dt\right| + \left|\int_3^5 (t^2 + 2t - 15)\,dt\right|$$
$$\left|\left[\tfrac{1}{3}t^3 + t^2 - 15t\right]\Big|_0^3\right| + \left|\left[\tfrac{1}{3}t^3 + t^2 - 15t\right]\Big|_3^5\right|$$
$$\left|(9 + 9 - 45) - 0\right| + \left|\left(\tfrac{125}{3} + 25 - 75\right) - (9 + 9 - 45)\right|$$
$$|-27| + \left|\tfrac{125}{3} - 23\right|$$
$$27 + \tfrac{56}{3} = \boxed{45.6667 \text{ meters}}$$

10. **Calculator active.** A particle's velocity is given by $v(t) = 6\cos 3t$, where t is measured in days, v is measured in yards per day, and $s(t)$ represents the particle's position.

a. If $s(0) = 5$, what is the value of $s\left(\frac{\pi}{2}\right)$? Calculator allowed.

$s(\frac{\pi}{2}) = 5 + \int_0^{\pi/2} 6\cos(3t)\,dt = \boxed{3 \text{ yards}}$

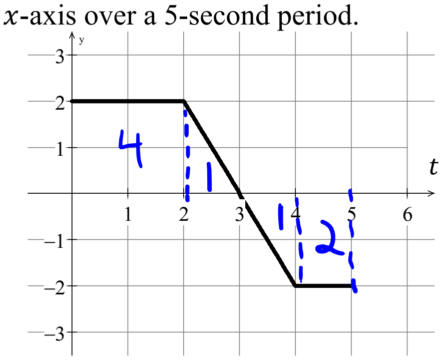
b. What is the net change in distance over the first $\frac{\pi}{2}$ days? Calculator allowed.

$\boxed{-2 \text{ yards}}$

c. What is the total distance traveled by the particle during the first $\frac{\pi}{2}$ days? Show the set up and use a calculator to find the answer.

$\int_0^{\pi/2} |6\cos(3t)|\,dt = \boxed{6 \text{ yards}}$

11. The graph to the right shows the velocity of an object moving along the x-axis over a 5-second period.



a. If the objected started 2 meters to the right, where is the object after 3 seconds?

$2 + \int_0^3 v(t)\,dt$

$2 + 5 = \boxed{7 \text{ meters right}}$

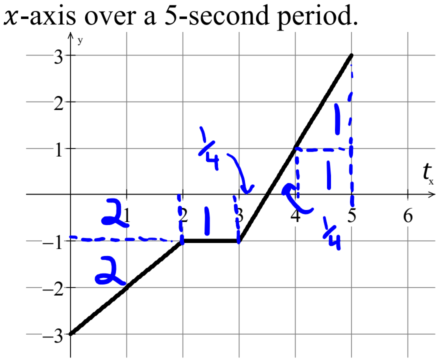
b. Where is the object after 5 seconds?

$2 + 5 - 3 = \boxed{4 \text{ meters right}}$

c. Find the total distance traveled by the object over the 5-second period.

$5 + 3 = \boxed{8 \text{ meters}}$

12. The graph to the right shows the **velocity** of an object moving along the x-axis over a 5-second period.



a. Find the total distance traveled by the object over the 5-second period.

$2 + 2 + 1 + \frac{1}{4} + \frac{1}{4} + 2 = \boxed{7.5}$

b. At time $t = 2$, the particle is at the point where $x = 10$. Where was the particle at time $t = 0$?

$10 - \int_0^2 v(t)\,dt = 10 - (-4)$

$\boxed{14}$

8.2 Connecting Pos, Vel, Acc with Integrals

Test Prep

13. **Calculator active.** At time t, $0 < t < 2.5$, the velocity of a particle moving along the x-axis is given by $v(t) = t\cos(t^2)$. Let $t = b$ be the time at which the particle changes direction from moving left to moving right. What is the total distance traveled by the particle during the time $0 < t < b$?

$b = 2.1708038$

$\int_0^b |v(t)|\,dt = 1.5$

(A) 0.5 (B) 1.253 (C) 1.5 (D) 2.171

This next problem is a common type of problem on an AP exam. Make sure you understand it!

14. **Calculator active.** Mr. Kelly and Mr. Sullivan are doing a morning speed-walk race going down a straight street. For $0 \le t \le 20$, Mr. Kelly's velocity at time t is given by $K(t) = \frac{16500}{t^2-5t+74.33}$ and Mr. Sullivan's velocity at time t is given by $S(t) = 41t^3e^{-0.6t}$. Both $K(t)$ and $S(t)$ are positive for $0 \le t \le 20$ and are measured in yards per minute, and t is measured in minutes. Mr. Kelly has a 5 yard head-start at $t = 0$, and is ahead of Mr. Sullivan for the entire time $0 \le t \le 20$.

a. Find the value of $\frac{1}{5}\int_{10}^{15} K(t)\,dt$. Using correct units, interpret the meaning of $\frac{1}{5}\int_{10}^{15} K(t)\,dt$ in the context of the problem.

$$\boldsymbol{\frac{1}{5}\int_{10}^{15} K(t)\,dt = 99.83242346}$$

Mr. Kelly's average velocity over the time interval $\boldsymbol{10 \le t \le 15}$ is $\boldsymbol{99.832}$ yards per minute.

b. At time $t = 7$, is Mr. Kelly speeding up or slowing down? Give a reason for your answer.

$$\boldsymbol{K(7) > 0 \text{ and } K'(7) = -19.03312 < 0}$$

At time $\boldsymbol{t = 7}$, Mr. Kelly is slowing down because he has positive velocity and negative acceleration.

c. Is the distance between Mr. Kelly and Mr. Sullivan at time $t = 7$ increasing or decreasing? Give a reason for your answer.

Mr. Kelly's velocity at $\boldsymbol{t = 7}$ is $\boldsymbol{K(7) = 186.7995019}$
Mr. Sullivan's velocity at $\boldsymbol{t = 7}$ is $\boldsymbol{S(7) = 210.8827968}$
Since $\boldsymbol{K(7) - S(7) < 0}$, the distance between Kelly and Sullivan at $\boldsymbol{t = 7}$ is decreasing.

d. What is the maximum distance between Mr. Kelly and Mr. Sullivan over the time interval $0 \le t \le 20$? Justify your answer.

The distance between Kelly and Sullivan at time $\boldsymbol{t}$ is given by $\boldsymbol{5 + \int_0^t K(x)\,dt - \int_0^t S(x)\,dx}$.

$$\boldsymbol{\frac{d}{dt}\left[5 + \int_0^t K(x)\,dt - \int_0^t S(x)\,dx\right] = K(t) - S(t) = 0}$$

$$\boldsymbol{t = 3.9268073 \text{ and } t = 8.246953}$$

t	Distance between Kelly and Sullivan
0	**5**
3.9268073	533.0478098
8.246953	428.7898849
20	959.6664322

The maximum distance between Kelly and Sullivan is $\boldsymbol{959.666}$ yards.