8.3 Applying Accumulation and Integrals
Calculus

Practice

Solutions

1. Rate of consumption of oil in the United States during the 1980s (in billions of barrels per year) is modeled by the function

$$R(t) = 27.08e^{\frac{t}{25}}$$

where t is the number of years after January 1, 1980. Find the total consumption of oil in the United States during the 1980s.

$$\int_0^{10} R(t)\,dt = \boxed{332.965 \text{ billion barrels}}$$

2. The ocean depth near the shore is changing at a rate modeled by $R(t) = 3.0368\sin\left(\frac{\pi}{6}t\right)$, measured in feet per hour t hours after 10 A.M. If the depth is 10 feet at 10 A.M., how deep is the water at 1:00 p.m.?

$$10 + \int_0^{3} R(t)\,dt = \boxed{15.7998 \text{ feet}}$$

3. Tom Sawyer is painting a fence at a rate of $(200 - 4t)$ square feet per hour, where t is the number of hours since he started painting. If the fence is 800 square feet, how long will it take him to finish painting the fence? Round your answer to the nearest minute.

$$\int_0^{x} (200 - 4t)\,dt = 800$$

$$200t - 2t^2 \Big|_0^x = 800$$

$$200x - 2x^2 - [0] = 800$$

$$0 = 2(x^2 - 100x + 400)$$

Calculator → Find zeros

$x = 4.174$ hours

$\cdot 60$ minutes

$\boxed{250 \text{ hours}}$

4. The rate of gallons of gasoline used per kilometer by a car to travel x kilometers is modeled by $g(x) = 0.15 - 0.15e^{-\frac{x}{2}} - 0.075xe^{-\frac{x}{2}}$. If the car started with 36 gallons, how many gallons are left after driving 200 kilometers.

$$36 - \int_0^{200} g(x)\,dx = 36 - 29.4 = 6.6 \text{ gallons}$$

5. Rain is falling at a rate modeled by $R(t)$ measured in inches per hour and t is measured in hours since noon. By noon, there has been 2 inches of rain that has already fallen that day. Write, but do not solve, an equation involving an integral to find the time A when the amount of rain that has fallen for the day has reached a total of 3 inches.

$$2 + \int_0^{A} R(t)\,dt = 3$$

6. Construction workers are pouring concrete at a rate modeled by $C(t)$ measured in cubic feet per minute and t is measured in minutes since the start of the workday. When the day begins, there was already 60 cubic feet of concrete that has been poured from the day before. Write, but do not solve, an equation involving an integral to find the time x when the amount of concrete poured has reached a total of 100 cubic feet.

$$60 + \int_0^{x} C(t)\,dt = 100$$

7. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. A table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, is shown below.

t (minutes)	$R(t)$ (gallons per minute)
0	15
20	60
30	45
50	60
60	70
90	75

a. Using correct units, interpret the meaning of $\int_0^{30} R(t)\,dt$ in the context of this problem.

The number of gallons of fuel consumed by the airplane during the first 30 minutes of the flight.

b. Use a left-point Riemann sum, with five subintervals, to approximate $\int_0^{90} R(t)\,dt$. Show the computations that lead to your answer. Indicate units of measure.

$$(20)(15) + (10)(60) + (20)(45) + (10)(60) + (30)(70)$$

$$300 + 600 + 900 + 600 + 2{,}100$$

$$\boxed{4{,}500 \text{ gallons}}$$

c. If the plane held 20,500 gallons of fuel when the flight started, use your answer from part (b) to estimate the amount of fuel remaining after 90 minutes.

$$\boxed{16{,}000 \text{ gallons}}$$

t (minutes)	0	10	20	30
$W(t)$ (°F)	100	89	81	75

8. The temperature of water in a bathtub at time t is modeled by a strictly decreasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. The water is cooling for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t are given in the table above. Use the data from the table to evaluate $\int_0^{30} W'(t)\,dt$, and explain the answer in the context of the problem.

$$W(t)\Big|_0^{30} = W(30) - W(0) = 75 - 100 = -25$$

After 30 minutes, the temperature of the water has decreased by 25 °F.

9. The rate of decay, in grams per minute, of a radioactive substance is a differentiable, decreasing function R of time, t, in years. The table below shows the decay rate if the substance has 3,000 grams, as recorded every 4 years over a 24-year period.

t (years)	0	4	8	12	16	20	24
$R(t)$ (grams per year)	100	95	70	80	72	65	70

a. Using correct units, interpret the meaning of $\int_0^{12} R(t)\,dt$ in the context of this problem.

The number of grams that have decayed of a radioactive substance over the first 12 years.

b. Use a mid-point Riemann sum, with three subintervals, to approximate $\int_0^{24} R(t)\,dt$. Show the computations that lead to your answer. Indicate units of measure.

$(8)(95) + (8)(80) + (8)(65)$

$760 + 640 + 520$

1920 grams

c. Using your answer from part (b), how much of the radioactive substance is left after 24 years?

$3000 - 1920$

1080 grams

10. **Calculator active problem.** Wild boars enter a community in Georgia at a rate modeled by the function $E(t) = 200 + 40\sin\left(\frac{\pi t}{6}\right)$. The boars leave the community at a rate modeled by the function

$$L(t) = 30 + 1.7^{0.2t^2}$$

Both $E(t)$ and $L(t)$ are measured in boar per year, and t is measured in years since 2010 ($t = 0$).

a. How many boars enter the community over the six-year period from 2010 to 2016.? Give your answer to the nearest whole number.

$$\int_0^6 E(t)\,dt = 1352.7887$$ 1353 boars

b. What is the average number of boars that leave the community per year over the 6-year period from 2010 to 2016?

$$\frac{1}{6}\int_0^6 L(t)\,dt = 37.257$$ boars per year

c. At what time t, for $0 \le t \le 8$, is the greatest number of boars in the community? Justify your answer.

$$A(t) = \int_0^t E(x)\,dx - \int_0^t L(x)\,dx$$

Let $A(t)$ be the # of boars gained or lost.

$$A'(t) = E(t) - L(t)$$

$$E(t) - L(t) = 0$$

$$E(t) = L(t)$$

$$t \simeq 6.8811981$$

t	$A(t)$
0	0
6.8811981	1194.144
8	896.2138

The greatest number of boars in the community is at $t = 6.881$.

d. Was the rate of change in the number of boars in the community increasing or decreasing in 2015 ($t = 5$)? Explain your reasoning.

$$E'(5) - L'(5) = -33.206 < 0$$

Because $E'(5) - L'(5) < 0$, the rate of change of the number of boars in the community is decreasing at time $t = 5$.