8.6 Area – More than Two Intersections

Calculus

Solutions

Practice

The given functions create boundaries for multiple regions.

1. $y = 2x^3 - x^2 - 7x,\ y = x^2 + 5x$

   a. Find x-values of the points of intersection, and label them from smallest to largest as A, B, and C.

   $A = \mathbf{-2}$

   $B = \mathbf{0}$

   $C = \mathbf{3}$

   b. Set up integrals that represent the area.

$$\int_{-2}^{0} (2x^3 - 2x^2 - 12x)\,dx + \int_{0}^{3} (-2x^3 + 2x^2 + 12x)\,dx$$

2. $y = x + e^{x^2-3x},\ y = 2x^2 - 4x + 2$

   a. Find x-values of the points of intersection, and label them from smallest to largest as A, B, C, and D.

   $A = \mathbf{-0.444007}$

   $B = \mathbf{0.3872903}$

   $C = \mathbf{2.0459255}$

   $D = \mathbf{3.6407356}$

   b. Set up integrals that represent the area.

$$\int_{A}^{B} \left(2x^2 - 5x + 2 - e^{x^2-3x}\right) dx + \int_{B}^{C} \left(5x + e^{x^2-3x} - 2x^2 - 2\right) dx + \int_{C}^{D} \left(2x^2 - 5x + 2 - e^{x^2-3x}\right) dx$$

3. $y = 6x - x^2, y = 5, x = 0$

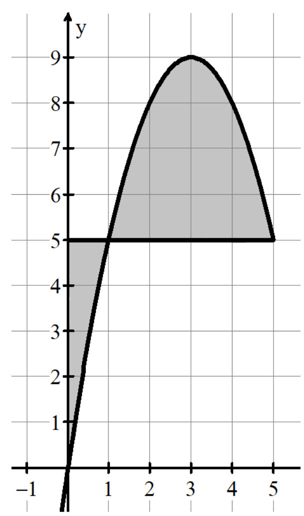
   a. Find x-values of the points of intersection, and label them from smallest to largest as A, B, and C.

   $A = \mathbf{0}$

   $B = \mathbf{1}$

   $C = \mathbf{5}$

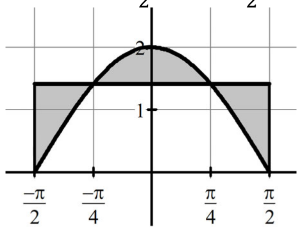
   b. Set up integrals that represent the area.



$$\int_{0}^{1} (5 - 6x + x^2)\,dx + \int_{1}^{5} (6x - x^2 - 5)\,dx$$

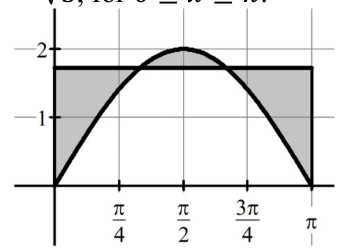
Write a set of integrals that represents the sum of all the areas of the shaded regions. Use exact values for your boundaries, not rounded decimals.

4. The figure shows the graph of $y = 2\cos(x)$, and the line $y = \sqrt{2}$, for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.



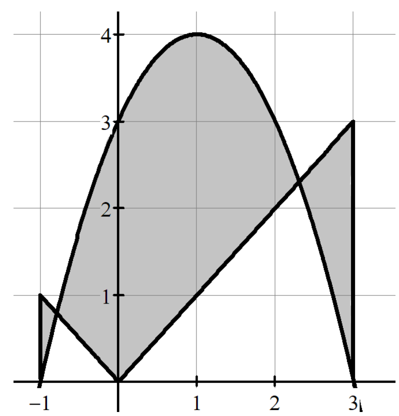
$$\int_{-\frac{\pi}{2}}^{-\frac{\pi}{4}} \left(\sqrt{2} - 2\cos x\right) dx + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(2\cos x - \sqrt{2}\right) dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\sqrt{2} - 2\cos x\right) dx$$

5. The figure shows the graph of $y = 2\sin(x)$, and the line $y = \sqrt{3}$, for $0 \le x \le \pi$.



$$\int_{0}^{\frac{\pi}{3}} \left(\sqrt{3} - 2\sin x\right) dx + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \left(2\sin x - \sqrt{3}\right) dx + \int_{\frac{2\pi}{3}}^{\pi} \left(\sqrt{3} - 2\sin x\right) dx$$

6. The figure shows the graphs of $y = |x|$ and $y = 3 + 2x - x^2$ for $-1 \le x \le 3$. The x-coordinates of the points of intersection of the graphs are x_1 and x_2, where $x_1 < x_2$. Write a sum of integrals that represents the shaded regions. You do NOT need to solve for x_1 and x_2.



$$\int_{-1}^{x_1} \left(x^2 - 3x - 3\right) dx + \int_{x_1}^{0} \left(-x^2 + 3x + 3\right) dx + \int_{0}^{x_2} \left(-x^2 + x + 3\right) dx + \int_{x_2}^{3} \left(x^2 - x - 3\right) dx$$

8.6 Area – More than Two Intersections

Test Prep

7. **Calculator active.** Let R be the region bounded by the graph of $y = e^{3x - x^2}$ and the horizontal lines $y = 1$ and $y = 3$, as shown in the figure below. Find the area of this bounded region.

$$A = 0.4269726$$
$$B = 2.5730274$$

$$\int_{0}^{A} \left(e^{3x - x^2} - 1\right) dx + \int_{A}^{B} (3 - 1)\, dx + \int_{B}^{3} \left(e^{3x - x^2} - 1\right) dx$$

$$\textbf{Area} \approx 5.0399$$