

8.7 Volumes with Cross Sections: Squares and Rectangles

CA #1

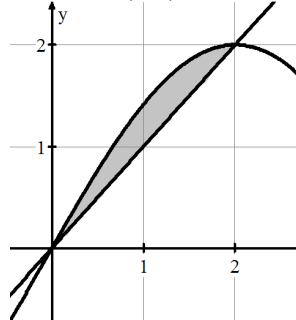
Calculus

Name: _____

The bounded region shown for each problem represents the base of a solid. Find the volume of each solid based on the given cross sections. Set up the integral(s) first, then use a calculator to evaluate.

1. Square cross sections perpendicular to the x -axis.

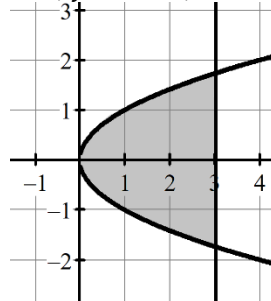
$$y = 2 \sin\left(\frac{\pi}{4}x\right) \text{ and } y = x$$



2. Square cross sections perpendicular to the y -axis.

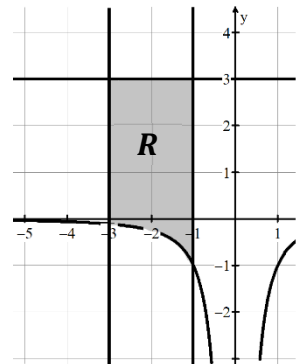
3. Square cross sections perpendicular to the x -axis.

$$y = \sqrt{x}, y = -\sqrt{x}, \text{ and } x = 3$$

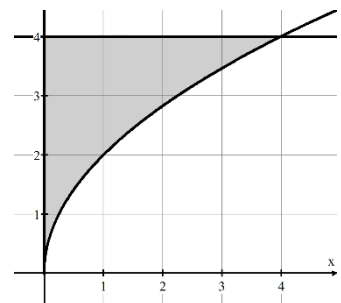


4. Square cross sections perpendicular to the y -axis.

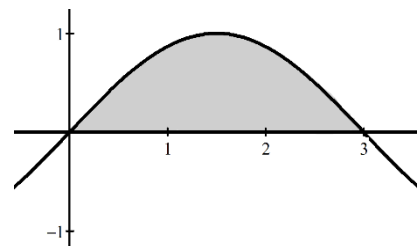
5. Let R be the region bounded by the graphs $y = -\frac{1}{x^2}$, $y = 3$, $x = -3$, and $x = -1$ as shown in the figure. The cross sections perpendicular to the x -axis are rectangles whose height is twice the width.



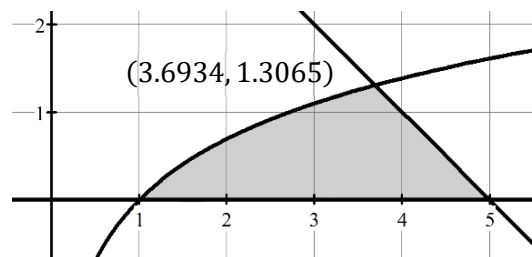
6. The base of a solid is the region bounded by the y -axis, the graph of $y = 2\sqrt{x}$ and the horizontal line $y = 4$. For the solid, each cross section perpendicular to the y -axis is a rectangle whose height is 3.



7. $y = \sin\left(\frac{\pi}{3}x\right)$ and the x -axis as shown in the figure. Each cross section perpendicular to the x -axis is a rectangle whose height is 4 times its width. What is the volume?



8. The x -axis $y = \ln x$, $y = 0$, and $y = 5 - x$. Each cross section perpendicular to the y -axis is a rectangle whose height is 6 times its width. What is the volume?



9. The graphs of $y = x^2 - 4$ and $y = 4 - 2x$ create a bounded region that represents the base of a solid. The cross sections of this solid are perpendicular to the x -axis and form squares. Find the volume of the solid.

Answers to 8.7 CA #1

1. $\int_0^2 \left(2 \sin\left(\frac{\pi}{4}x\right) - x\right)^2 dx \approx 0.182$	2. $\int_0^2 \left(y - \frac{4}{\pi} \sin^{-1}\left(\frac{y}{2}\right)\right)^2 dy \approx 0.182$	3. $\int_0^3 (2\sqrt{x})^2 dx = 18$
4. $\int_{-\sqrt{3}}^{\sqrt{3}} (3 - y^2)^2 dy = 16.6276$	5. $\int_{-3}^{-1} 2 \left(3 + \frac{1}{x^2}\right)^2 dx \approx 44.6419$	6. $\int_0^4 3 \left(\frac{y^2}{4}\right) dx = 16$
7. $\int_0^3 4 \left(\sin\left(\frac{\pi}{3}x\right)\right)^2 dx = 6$	8. $\int_0^{1.3065} 6(5 - y - e^y)^2 dy \approx 51.1368$	9. $\int_{-4}^2 (-x^2 - 2x + 8)^2 dx = 259.2$