## Calculus

Write your questions and thoughts here!
8.7 Volumes with Cross Sections: Squares and Rectangles

1. A region is bounded by $y=x^{2}$ and $y=\sqrt{x}$, and forms the base of a solid. For this solid, each cross section perpendicular to the $x$-axis is a square. What is the volume of the solid?


## Volume of a Solid with known Cross Sections

$$
V=
$$

where
is the
of a cross section perpendicular to the $x$-axis.

SQUARE cross sections

$$
V=\int_{a}^{b}
$$

where $s=$

RECTANGLE cross sections

$$
V=\int_{a}^{b}
$$

where width $=$
height $=$ given in the problem
2. Same base as \#1, but the square cross sections are taken perpendicular to the $\boldsymbol{y}$-axis.

3. The base of a solid is bounded by $y=x^{3}, y=0$, and $x=2$.
a. Find the volume if the cross sections, taken perpendicular to the $x$-axis, form a rectangle whose height is 2 times its width.
b. Find the volume if the cross sections, taken perpendicular to the $y$-axis, form a rectangle whose height is 6 .


Cross sections when you have no graph.
4. The graphs of $y=x^{2}-4$ and $y=2 x-x^{2}$ create a bounded area that is the base of a solid. This solid has cross sections that are perpendicular to the $x$-axis and form squares.

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## Practice

The bounded region shown for each problem represents the base of a solid. Find the volume of each solid based on the given cross sections. Set up the integral(s) first, then use a calculator to evaluate.

1. Square cross sections perpendicular to the $x$-axis.

2. Square cross sections
perpendicular to the $y$-axis.
3. Square cross sections perpendicular to the $x$-axis.

4. Square cross sections perpendicular to the $y$-axis.
5. Square cross sections perpendicular to the $x$-axis.

6. Square cross sections perpendicular to the $y$-axis.
7. The $y$-axes, $y=\sin x$, and $y=1$ for $0 \leq x \leq \frac{\pi}{2}$. Each cross section perpendicular to the $x$-axis is a rectangle whose height is 3 times its width.

8. The region in the first quadrant bounded by $y=e^{x}$ and the vertical line $x=1$. The cross sections perpendicular to the $y$-axis are rectangles whose height is 2 times their width. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

9. $y=\sqrt{x}$ and $y=\frac{x}{3}$ cross sections perpendicular to the $y$-axis are rectangles whose height is 6 .

10. The $x$-axis and the graph of $y=\sqrt{1-x^{2}}$. Each cross section perpendicular to the $x$-axis is a rectangle whose height is 10 times the width.


The following curves create a bounded region. Each solid has cross sections perpendicular to the x-axis that are squares. Find the volume of each solid based on the given cross sections. Set up the integral(s) first, then use a calculator to evaluate.
11. $y=x-4, y=4-x$, and $x=0 . \quad$ 12. $x^{2}+y^{2}=100$
13. $y=x^{2}-4$, and $y=4$

No test prep for this lesson because these questions are similar to the free response portion of an AP Exam.

