8.7 Volumes with Cross Sections: Squares and Rectangles

Calculus

Solutions

Practice

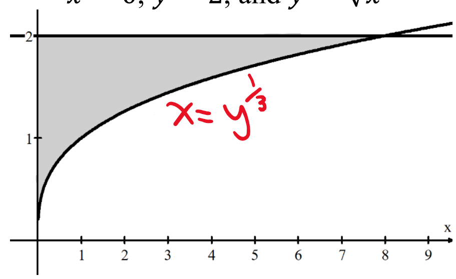
The bounded region shown for each problem represents the base of a solid. Find the volume of each solid based on the given cross sections. Set up the integral(s) first, then use a calculator to evaluate.

1. Square cross sections perpendicular to the x-axis.

$$\int_0^8 \left(2-\sqrt[3]{x}\right)^2 dx$$

3.2

$x = 0$, $y = 2$, and $y = \sqrt[3]{x}$



2. Square cross sections perpendicular to the y-axis.

$$\int_0^2 \left(y^3\right)^2 dy$$

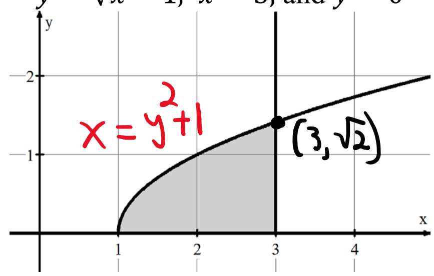
18.2857

3. Square cross sections perpendicular to the x-axis.

$$\int_1^3 (x-1)\, dx$$

2

$y = \sqrt{x-1}$, $x = 3$, and $y = 0$



4. Square cross sections perpendicular to the y-axis.

$$\int_0^{\sqrt{2}} \left(3-(y^2+1)\right)^2 dy$$

$$\int_0^{\sqrt{2}} \left(2-y^2\right)^2 dy$$

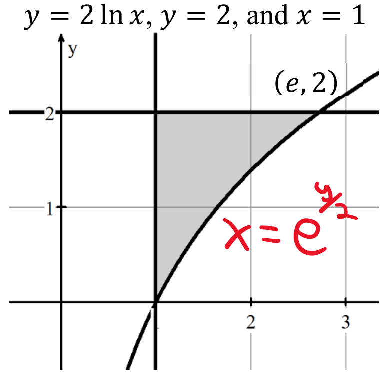
3.0169

5. Square cross sections perpendicular to the x-axis.

$$\int_1^e \left(2-2\ln x\right)^2 dx$$

1.746

$y = 2\ln x$, $y = 2$, and $x = 1$

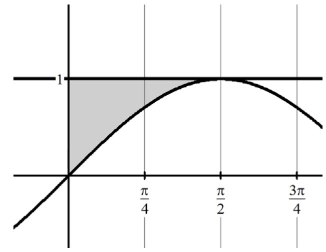


6. Square cross sections perpendicular to the y-axis.

$$\int_0^2 \left(e^{y/2}-1\right)^2 dy$$

1.5159

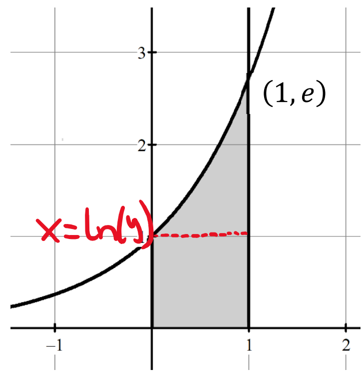
7. The y-axes, $y = \sin x$, and $y = 1$ for $0 \le x \le \frac{\pi}{2}$. Each cross section perpendicular to the x-axis is a rectangle whose height is 3 times its width.



$$\int_0^{\pi/2} (1-\sin x)(3)(1-\sin x)\,dx$$

$$3\int_0^{\pi/2} (1-\sin x)^2\,dx = 1.0685$$

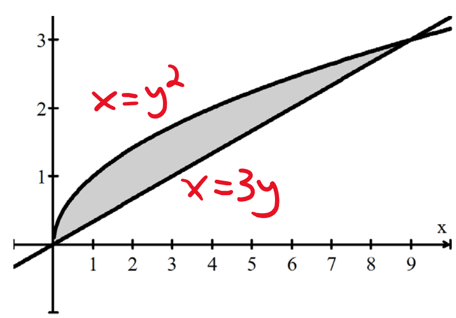
8. The region in the first quadrant bounded by $y = e^x$ and the vertical line $x = 1$. The cross sections perpendicular to the $\boxed{y\text{-axis}}$ are rectangles whose height is 2 times their width. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.



$$\int_0^1 (1)\cdot(2)(1)\,dy + \int_1^e (1-\ln y)\cdot(2)(1-\ln y)$$

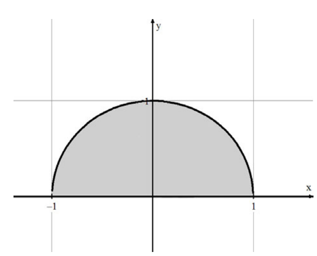
$$2\int_0^1 dy + 2\int_1^e (1-\ln y)^2\,dy$$

9. $y = \sqrt{x}$ and $y = \frac{x}{3}$ cross sections perpendicular to the $\boxed{y\text{-axis}}$ are rectangles whose height is 6.



$$\int_0^3 (3y-y^2)\cdot 6\,dy = 27$$

10. The x-axis and the graph of $y = \sqrt{1-x^2}$. Each cross section perpendicular to the x-axis is a rectangle whose height is 10 times the width.



$$\int_{-1}^1 (\sqrt{1-x^2})\cdot(10)(\sqrt{1-x^2})\,dx$$

$$10\int_{-1}^1 (1-x^2)\,dx = 13.333$$

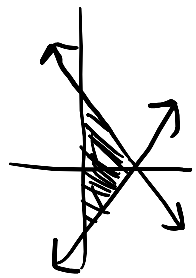
The following curves create a bounded region. Each solid has cross sections perpendicular to the x-axis that are squares. Find the volume of each solid based on the given cross sections. Set up the integral(s) first, then use a calculator to evaluate.

11. $y = x - 4,\ y = 4 - x$, and $x = 0$.

$x - 4 = 4 - x$

$2x = 8$

$x = 4$



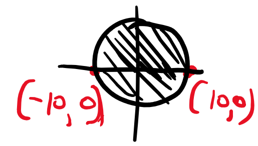
$\int_0^4 [(4-x)-(x-4)]^2\,dx$

$\int_0^4 (8-2x)^2\,dx$

85.333

12. $x^2 + y^2 = 100$

Circle!



$y^2 = 100 - x^2$

$y = \pm\sqrt{100-x^2}$

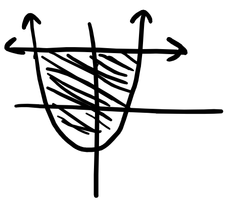
$\int_{-10}^{10} \left(\sqrt{100-x^2} - -\sqrt{100-x^2}\right)^2 dx$

$\int_{-10}^{10} \left(2\sqrt{100-x^2}\right)^2 dx$

$4\int_{-10}^{10} (100-x^2)\,dx$

5333.333

13. $y = x^2 - 4$, and $y = 4$



$x^2 - 4 = 4$

$x^2 = 8$

$x = \pm\sqrt{8}$

$\int_{-\sqrt{8}}^{\sqrt{8}} (4-(x^2-4))^2\,dx$

$\int_{-\sqrt{8}}^{\sqrt{8}} (8-x^2)^2\,dx$

193.087

No test prep for this lesson because these questions are similar to the free response portion of an AP Exam.