8.8 Volumes with Cross Sections: Triangles and Semicircles

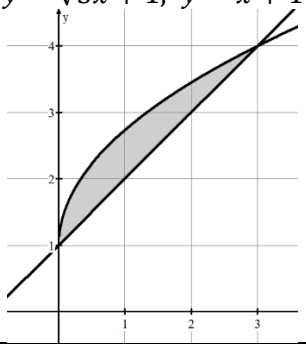
CA #1

Calculus Name: _____

The bounded region shown for each problem represents the base of a solid. Find the volume of each solid based on the given cross sections. Set up the integral(s) first, then use a calculator to evaluate.

1. Semicircle cross sections perpendicular to the x-axis.

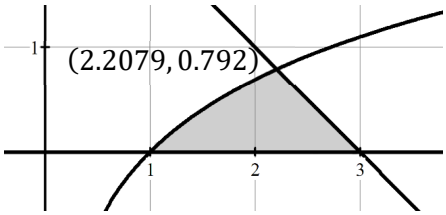
$y = \sqrt{3x} + 1,\ y = x + 1$



2. Equilateral triangle cross sections perpendicular to the y-axis.

3. Isosceles right triangle cross sections perpendicular to the x-axis.

$y = \ln x, y = 3 - x$ and the x-axis



4. Semicircle cross sections perpendicular to the y-axis.

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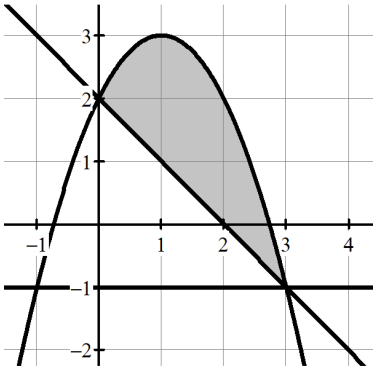
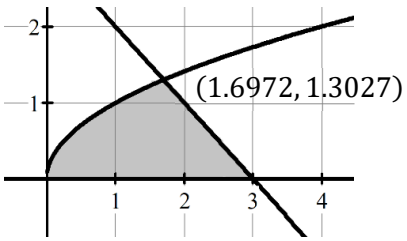
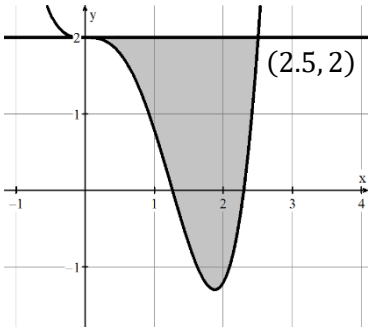
5. A region is bounded by $y = -x^2 + 2x + 3$ and $y = 2 - x$ as shown in the figure. The cross sections perpendicular to the x-axis are isosceles right triangles. Set up the integral, but do not evaluate.



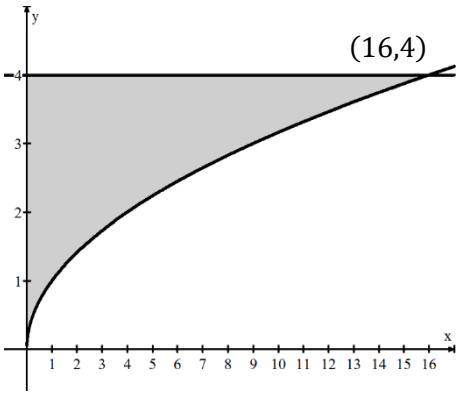
6. The base of a solid is the region bounded by the y-axis, the graphs of $y = \sqrt{x},\ y = 0$, and $y = 3 - x$. For the solid, each cross section perpendicular to the y-axis is a semicircle. Set up the integral, but do not evaluate.



7. A region is bounded by $y = 0.8x^4 - 2x^3 + 2$ and $y = 2$ as shown in the figure. Each cross section perpendicular to the x-axis is an equilateral triangle. Set up the integral, but do not evaluate.



8. The region bounded by the y-axis, the graph of $y = \sqrt{x}$ and the line $y = 4$ is shown. For the solid, each cross section perpendicular to the y-axis is a semicircle. Set up the integral, but do not evaluate.



9. The graphs of $y = x^2 - x - 3$ and $y = x$ create a bounded region that represents the base of a solid. The cross sections of this solid are perpendicular to the x-axis and form semicircles. Find the volume of the solid. Set up the integral, but do not evaluate.

Answers to 8.8 CA #1

1. $\frac{\pi}{8}\int_0^3 \left(\sqrt{3x} - x\right)^2 dx \approx 0.353$	2. $\int_1^{\frac{4\sqrt{3}}{4}} \left(y - 1 - \frac{(y-1)^2}{3}\right)^2 dy \approx 0.3897$	3. $\frac{1}{2}\int_1^{2.2079} (\ln x)^2\, dx + \frac{1}{2}\int_{2.2079}^3 (3-x)^2\, dx \approx 0.2345$
4. $\frac{\pi}{8}\int_0^{0.792} (3 - y - e^y)^2\, dy \approx 0.4648$	5. $\frac{1}{2}\int_0^3 (-x^2 + 3x + 1)^2\, dx$	6. $\frac{\pi}{8}\int_0^{1.3027} (3 - y - y^2)^2\, dy$
7. $\frac{\sqrt{3}}{4}\int_0^{2.5} (-0.8x^4 + 2x^3)^2\, dx$	8. $\frac{\pi}{8}\int_0^4 (y^4)\, dy$	9. $\frac{\pi}{8}\int_{-1}^3 (-x^2 + 2x + 3)^2\, dx$