8.8 Volumes with Cross Sections: Triangles and Semicircles

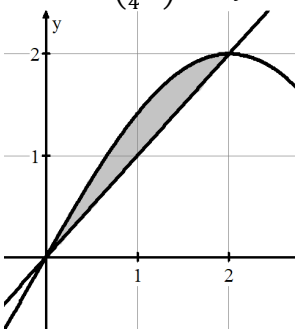
CA #2

Calculus Name: _____

The bounded region shown for each problem represents the base of a solid. Find the volume of each solid based on the given cross sections. Set up the integral(s) first, then use a calculator to evaluate.

1. Equilateral triangle cross sections perpendicular to the x-axis.

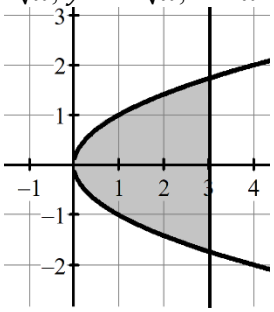
$y = 2\sin\left(\frac{\pi}{4}x\right)$ and $y = x$



2. Semicircle cross sections perpendicular to the y-axis.

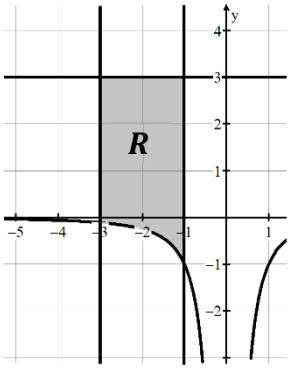
3. Semicircle cross sections perpendicular to the x-axis.

$y = \sqrt{x}$, $y = -\sqrt{x}$, and $x = 3$

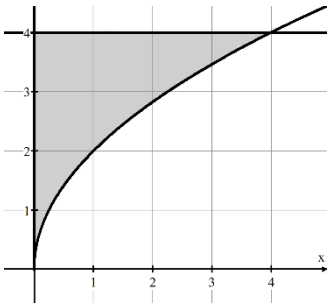


4. Isosceles right triangle cross sections perpendicular to the y-axis.

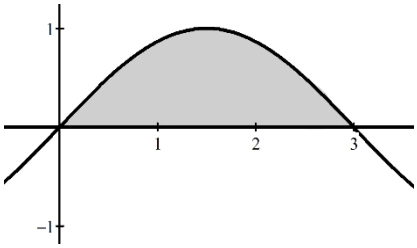
5. Let R be the region bounded by the graphs $y = -\frac{1}{x^2}$, $y = 3$, $x = -3$, and $x = -1$ as shown in the figure. The cross sections perpendicular to the x-axis are semicircles. Set up the integral to find the volume of the solid, but do not evaluate.



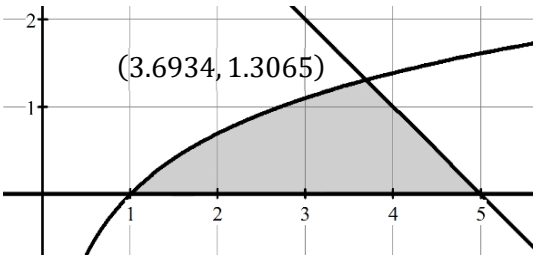
6. The base of a solid is the region bounded by the y-axis, the graph of $y = 2\sqrt{x}$ and the horizontal line $y = 4$. For the solid, each cross section perpendicular to the y-axis is an isosceles right triangle. Set up the integral to find the volume of the solid, but do not evaluate.



7. $y = \sin\left(\frac{\pi}{3}x\right)$ and the x-axis as shown in the figure. Each cross section perpendicular to the x-axis is a semicircle. Set up the integral to find the volume of the solid, but do not evaluate.



8. The x-axis $y = \ln x$, $y = 0$, and $y = 5 - x$. Each cross section perpendicular to the y-axis is an equilateral triangle. Set up the integral to find the volume of the solid, but do not evaluate.



9. The graphs of $y = x^2 - 4$ and $y = 4 - 2x$ create a bounded region that represents the base of a solid. The cross sections of this solid are perpendicular to the x-axis and form semicircles. Set up the integral to find the volume of the solid, but do not evaluate.

Answers to 8.8 CA #2

1. $\frac{\sqrt{3}}{4}\int_0^2 \left(2\sin\left(\frac{\pi}{4}x\right) - x\right)^2 dx \approx 0.0788$	2. $\frac{\pi}{8}\int_0^2 \left(y - \frac{4}{\pi}\sin^{-1}\left(\frac{y}{2}\right)\right)^2 dy \approx 0.0715$	3. $\frac{\pi}{8}\int_0^3 \left(2\sqrt{x}\right)^2 dx \approx 7.0685$
4. $\frac{1}{2}\int_{-\sqrt{3}}^{\sqrt{3}} (3 - y^2)^2\, dy = 8.1318$	5. $\frac{\pi}{8}\int_{-3}^{-1} 2\left(3 + \frac{1}{x^2}\right)^2 dx$	6. $\frac{1}{2}\int_0^4 \left(\frac{y^2}{4}\right)^2 dx$
7. $\frac{\pi}{8}\int_0^3 \left(\sin\left(\frac{\pi}{3}x\right)\right)^2 dx$	8. $\frac{\sqrt{3}}{4}\int_0^{1.3065} (5 - y - e^y)^2\, dy$	9. $\frac{\pi}{8}\int_{-4}^2 (-x^2 - 2x + 8)^2\, dx$