Calculus

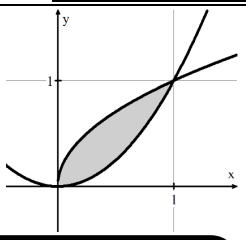
Write your questions and thoughts here!

8.8 Volumes with Cross Sections: Triangles and Semicircles

Notes

A region is bounded by $y = x^2$ and $y = \sqrt{x}$, and forms the base of various solids. The cross sections of each solid are listed below, but they are all perpendicular to the x-axis.

a. Equilateral triangle
b. Isosceles right triangle
c. Semicircle



Volume of a Solid with known Cross Sections

$$V = \int_a^b A(x)\,dx$$

where $\boldsymbol{A(x)}$ is the **area** of a cross section perpendicular to the x-axis.

EQUILATERAL TRIANGLE cross sections	**ISOSCELES RIGHT TRIANGLE** cross sections
$V = \int_a^b$ where $s =$	$V = \int_a^b$ $s =$

SEMICIRCLES cross sections

$$V = \int_a^b$$

where $r =$

Find the volume of each solid with the given cross sections perpendicular to the x-axis.		
a. Equilateral triangle	b. Isosceles right triangle	c. Semicircle

8.8 Volumes with Cross Sections: Triangles and Semicircles

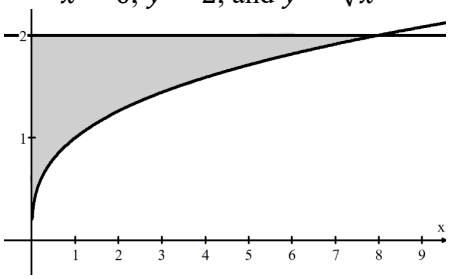
Calculus

Practice

The bounded region shown for each problem represents the base of a solid. Find the volume of each solid based on the given cross sections. Set up the integral(s) first, then use a calculator to evaluate.

1. Semicircle cross sections perpendicular to the x-axis.

$x = 0$, $y = 2$, and $y = \sqrt[3]{x}$



2. Equilateral triangle cross sections perpendicular to the y-axis.

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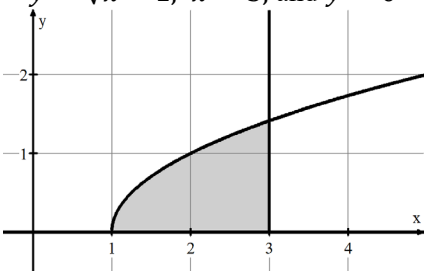
3. Isosceles right triangle, with a leg on the base perpendicular to the x-axis.

$y = \sqrt{x-1}$, $x = 3$, and $y = 0$



4. Semicircle cross sections perpendicular to the y-axis.

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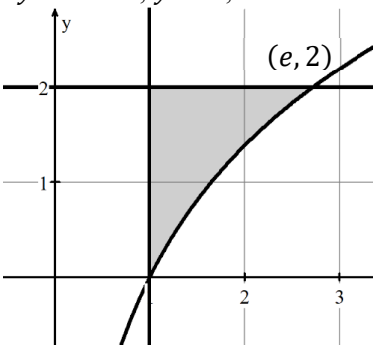
5. Semicircle cross sections perpendicular to the x-axis.

$y = 2\ln x$, $y = 2$, and $x = 1$



6. Isosceles right triangle, with a leg on the base perpendicular to the y-axis.

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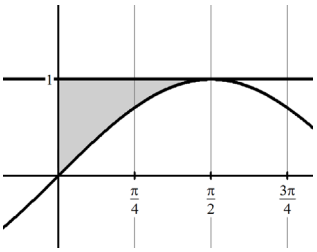
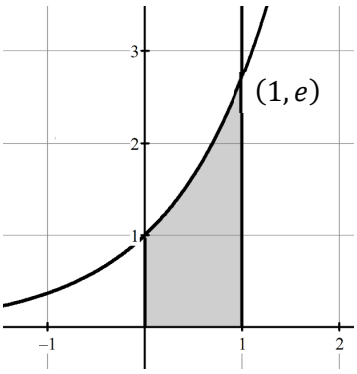
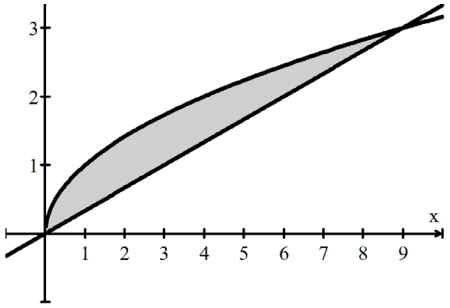
7. The y-axis, $y = \sin x$, and $y = 1$ for $0 \le x \le \frac{\pi}{2}$. Each cross section perpendicular to the y-axis is a semicircle.



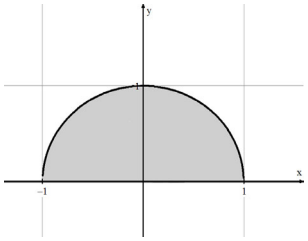
8. The region in the first quadrant bounded by $y = e^x$ and the vertical line $x = 1$. The cross sections perpendicular to the y-axis are equilateral triangles.



9. The region bounded by $y = \sqrt{x}$ and $y = \frac{x}{3}$ is the base of a solid with cross sections perpendicular to the y-axis that are semicircles.



10. The x-axis and the graph of $y = \sqrt{1 - x^2}$. Each cross-section perpendicular to the x-axis is an isosceles right triangle with the leg (not the hypotenuse) on the base.



The following curves create a bounded region. Each solid has cross sections perpendicular to the x-axis that are semicircles. Find the volume of each solid based on the given cross sections. Set up the integral(s) first, then use a calculator to evaluate.

11. $y = x - 4,\ y = 4 - x$, and $x = 0$.

12. $x^2 + y^2 = 49$

No test prep for this lesson because these questions are similar to the free response portion of an AP Exam.