8.8 Volumes with Cross Sections: Triangles and Semicircles

Calculus

Solutions

Practice

The bounded region shown for each problem represents the base of a solid. Find the volume of each solid based on the given cross sections. Set up the integral(s) first, then use a calculator to evaluate.

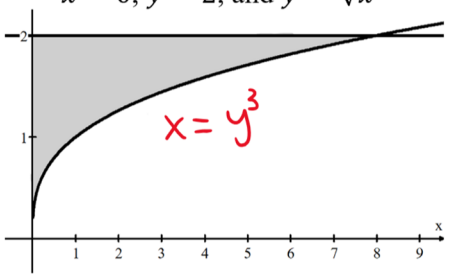
1. Semicircle cross sections perpendicular to the x-axis.

$$\int_0^8 \frac{\pi}{2}\left(\frac{2-\sqrt[3]{x}}{2}\right)^2 dx$$

$$\frac{\pi}{8}\int_0^8 \left(2-\sqrt[3]{x}\right)^2 dx$$

1.2566

$x = 0$, $y = 2$, and $y = \sqrt[3]{x}$



2. Equilateral cross sections perpendicular to the y-axis.

$$\int_0^2 \frac{\sqrt{3}}{4}\left(y^3\right)^2 dy$$

$$\frac{\sqrt{3}}{4}\int_0^2 y^6\, dy \simeq$$

7.9179

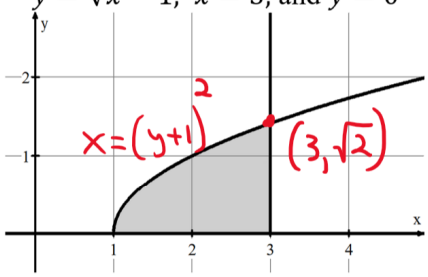
3. Isosceles right triangle perpendicular to the x-axis.

$$\int_1^3 \frac{1}{2}\left(\sqrt{x-1}\right)^2 dx$$

$$\frac{1}{2}\int_1^3 (x-1)\, dx$$

1

$y = \sqrt{x-1}$, $x = 3$, and $y = 0$



4. Semicircle cross sections perpendicular to the y-axis.

$$\int_0^{\sqrt{2}} \frac{\pi}{2}\left(\frac{3-(y^2+1)}{2}\right)^2 dx$$

$$\frac{\pi}{8}\int_0^{\sqrt{2}} \left(3-(y^2+1)\right)^2 dx$$

1.1847

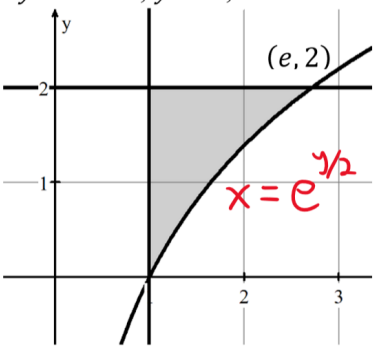
5. Semicircle cross sections perpendicular to the x-axis.

$$\int_1^e \frac{\pi}{2}\left(\frac{2-2\ln x}{2}\right)^2 dx$$

$$\frac{\pi}{2}\int_1^e (1-\ln x)^2 dx$$

0.6857

$y = 2\ln x$, $y = 2$, and $x = 1$



6. Isosceles cross sections perpendicular to the y-axis.

$$\int_0^2 \frac{1}{2}\left(e^{\frac{y}{2}}-1\right)^2 dy$$

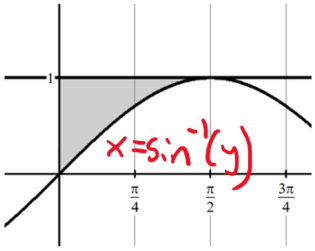
0.7579

7. The y-axis, $y = \sin x$, and $y = 1$ for $0 \le x \le \frac{\pi}{2}$. Each cross section perpendicular to the y-axis is a semicircle.

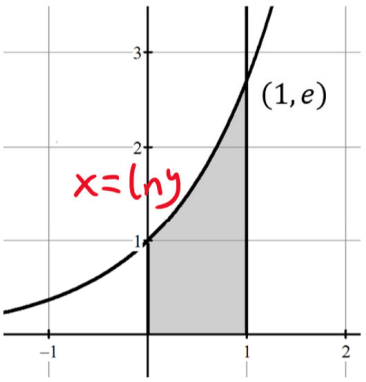
$$\int_0^1 \frac{\pi}{2}\left(\frac{\sin^{-1}(y)}{2}\right)^2 dy$$

$$\frac{\pi}{8}\int_0^1 \left(\sin^{-1}y\right)^2 dy \simeq$$

0.1835



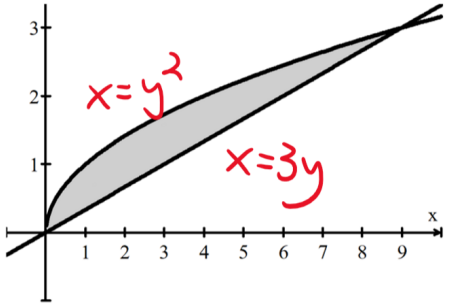
8. The region in the first quadrant bounded by $y = e^x$ and the vertical line $x = 1$. The cross sections perpendicular to the y-axis are equilateral triangles.



$$\int_0^1 \frac{\sqrt{3}}{4}(1)^2\,dy + \int_1^e \frac{\sqrt{3}}{4}(1-\ln y)^2\,dy$$

0.622

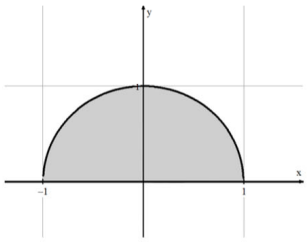
9. $y = \sqrt{x}$ and $y = \frac{x}{3}$ cross sections perpendicular to the y-axis are semicircles.



$$\int_0^3 \frac{\pi}{2}\left(\frac{3y-y^2}{2}\right)^2 dy$$

$$\frac{\pi}{8}\int_0^3 (3y-y^2)^2\,dy \simeq$$ **3.1808**

10. The x-axis and the graph of $y = \sqrt{1-x^2}$. Each cross section perpendicular to the x-axis is an isosceles triangle.

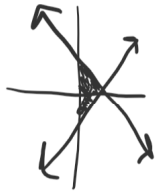


$$\int_{-1}^1 \frac{1}{2}\left(\sqrt{1-x^2}\right)^2 dx$$

$$\frac{1}{2}\int_{-1}^1 (1-x^2)\,dx = \frac{2}{3} \simeq$$ **0.6666**

The following curves create a bounded region. Each solid has cross sections perpendicular to the x-axis that are semicircles. Find the volume of each solid based on the given cross sections. Set up the integral(s) first, then use a calculator to evaluate.

11. $y = x - 4$, $y = 4 - x$, and $x = 0$.



$x - 4 = 4 - x$

$2x = 8$

$x = 4$

$$\int_0^4 \frac{\pi}{2}\left(\frac{4-x-(x-4)}{2}\right)^2 dx$$

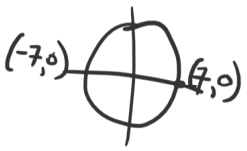
$$\frac{\pi}{8}\int_0^4 (8-2x)^2\,dx$$

33.510

12. $x^2 + y^2 = 49$

$y^2 = 49 - x^2$

$y = \pm\sqrt{49-x^2}$



$$\int_{-7}^7 \frac{\pi}{2}\left(\frac{\sqrt{49-x^2}-(-\sqrt{49-x^2})}{2}\right)^2 dx$$

$$\frac{\pi}{2}\int_{-7}^7 (49-x^2)\,dx$$

718.3775

No test prep for this lesson because these questions are similar to the free response portion of an AP Exam.