8.9 Disc Method: Revolve Around x or y Axis
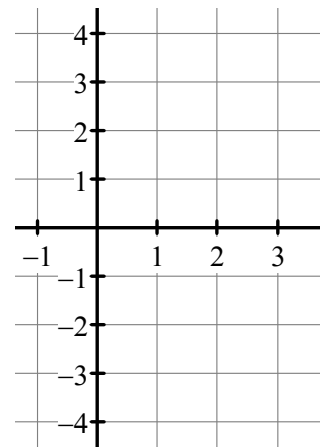
Notes

Finding the volume of a solid of revolution.

1. Sketch the area bounded by the equations. $y = x^2$, $y = 0, x = 2$.
2. Revolve it around the x-axis to create a solid.
3. What does the area of a cross section look like?

4. What is the area of a circle?

5. What is the radius of this circle?

6. What is the area of one cross-section?

7. What is the volume of the solid?

Volume of a Solid of Revolution

$$V = \int_a^b$$

where   is the "distance" between the axis of revolution and the outside of the solid.

2. Take the region bounded by $y = e^x$, $y = 0$, $x = 0$, and $x = 3$. Revolve this region about the x-axis. Find the volume of the solid formed.

3. Take the region bounded by $y = 4 - 2x$, $y = 0$, and $x = 0$. Revolve this region about the y-axis. Setup the integral to find the volume of the solid formed.

For each problem, sketch the area bounded by the equations and revolve it around the axis indicated. Find the volume of the solid formed by this revolution. Leave your answers in terms of π.

1. $y = -x + 2$, $x = 0$, $y = 0$. Revolve around the x-axis.

2. $y = \sqrt{x}$, $x = 1$, $x = 4$. Revolve around the x-axis.

3. $y = -\frac{1}{2}x + 2$, $x = 0$, $y = 0$. Revolve around the y-axis.

4. $y = 4 - x^2$, $y = 0$, $x \geq 0$. Revolve around the x-axis.

5. $y = e^x$, $x = 0$, $y = e$. Revolve around the y-axis. Setup, but **do not** evaluate.

6. $y = \sqrt{9 - x^2}$, $x \geq 0$, $y = 0$. Revolve around the x-axis. Setup, but **do not** evaluate.

7. $y = x^3$, $y = 0$, $x = 2$. Revolve around the x-axis. Setup, but **do not** evaluate.

8. $y = 4 - x^2$, $y = 0$, $x \geq 0$. Revolve around the y-axis. Setup, but **do not** evaluate.

9. $y = \sqrt{\sin x}$, $y = 0$, $x = 0$, $x = \pi$. Revolve around the x-axis. Setup, but **do not** evaluate.

10. $y = \sqrt{9 - x^2}$, $y = 0$, $x \geq 0$. Revolve around the y-axis. Setup, but **do not** evaluate.

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11. **Calculator allowed, but show your steps!** Mr. Brust is pouring his favorite liquid gelatin into a mixing bowl. The bowl's shape can be obtained by revolving the curve $y = \frac{9}{2401} x^4$ from $x = 0$ to $x = 7$ about the y-axis, where x and y are measured in centimeters. The gelatin is poured into the empty bowl at a constant rate of 14 cubic centimeters per second.

    Let h be the depth, in centimeters, of gelatin in the bowl. How fast was the depth of the gelatin in the bowl increasing when $h = 4$? Leave your answer in terms of π. Indicate all units of measure.