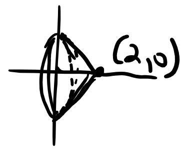
8.9 Disc Method: Revolve Around x or y Axis

Calculus

Practice

For each problem, sketch the area bounded by the equations and revolve it around the axis indicated. Find the volume of the solid formed by this revolution. Leave your answers in terms of π.

1. $y = -x + 2,\ x = 0,\ y = 0$. Revolve around the x-axis.



$\int_0^2 \pi(-x+2)^2\,dx$

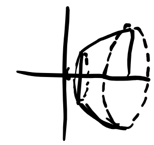
$\pi\int_0^2 (x^2-4x+4)\,dx$

$\pi\left[\frac{x^3}{3} - 2x^2 + 4x\right]\Big|_0^2$

$\pi\left[\left(\frac{8}{3} - 8 + 8\right) - (0)\right]$

$\pi\left(\frac{8}{3}\right) = \boxed{\frac{8}{3}\pi}$

2. $y = \sqrt{x},\ x = 1,\ x = 4$. Revolve around the x-axis.



$\int_1^4 \pi(\sqrt{x})^2\,dx$

$\pi\int_1^4 x\,dx$

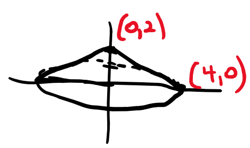
$\pi\left[\frac{x^2}{2}\right]\Big|_1^4$

$\pi\left[\frac{16}{2} - \frac{1}{2}\right]$

$\boxed{\frac{15}{2}\pi}$

3. $y = -\frac{1}{2}x + 2,\ x = 0,\ y = 0$. Revolve around the y-axis.

$x = 4 - 2y$



$\int_0^2 \pi(4-2y)^2\,dy$

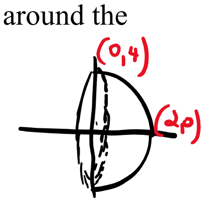
$\pi\int_0^2 (16 - 16y + 4y^2)\,dy$

$\pi\left[16y - 8y^2 + \frac{4}{3}y^3\right]\Big|_0^2$

$\pi\left[\left(32 - 32 + \frac{4}{3}(8)\right) - (0)\right]$

$\boxed{\frac{32}{3}\pi}$

4. $y = 4 - x^2,\ y = 0,\ x \geq 0$. Revolve around the x-axis.



$\int_0^2 \pi(4-x^2)^2\,dx$

$\pi\int_0^2 (16 - 8x^2 + x^4)\,dx$

$\pi\left[16x - \frac{8}{3}x^3 + \frac{x^5}{5}\right]\Big|_0^2$

$\pi\left[\left(32 - \frac{64}{3} + \frac{32}{5}\right) - (0)\right]$

Uhh... use a calculator to help!

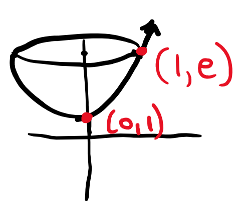
$\pi\left[\frac{480}{15} - \frac{320}{15} + \frac{96}{15}\right]$

$\pi\left[\frac{256}{15}\right] = \boxed{\frac{256}{15}\pi}$

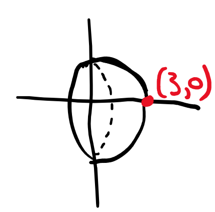
5. $y = e^x,\ x = 0,\ y = e$. Revolve around the y-axis. Setup, but **do not** evaluate.

$\ln y = x$

$\boxed{\int_1^e \pi(\ln y)^2\,dy}$

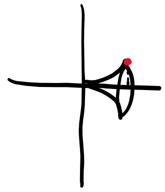


6. $y = \sqrt{9 - x^2},\ x \geq 0,\ y = 0$. Revolve around the x-axis. Setup, but **do not** evaluate.



$\boxed{\int_0^3 \pi(9-x^2)\,dx}$

7. $y = x^3,\ y = 0,\ x = 2$. Revolve around the x-axis. Setup, but **do not** evaluate.

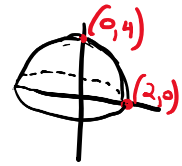


$$\boxed{\int_0^2 \pi x^6\, dx}$$

8. $y = 4 - x^2,\ y = 0,\ x \geq 0$. Revolve around the y-axis. Setup, but **do not** evaluate.

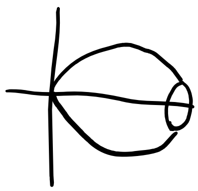
$x^2 = 4 - y$

$x = \pm\sqrt{4-y}$ $\quad x = \sqrt{4-y}$



$$\boxed{\int_0^4 \pi(4-y)\, dy}$$

9. $y = \sqrt{\sin x},\ y = 0,\ x = 0,\ x = \pi$. Revolve around the x-axis. Setup, but **do not** evaluate. You might need a calculator to help you see the graph.



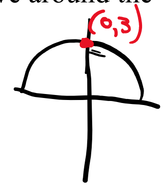
$$\boxed{\int_0^{\pi} \pi \sin x\, dx}$$

10. $y = \sqrt{9 - x^2},\ y = 0,\ x \geq 0$. Revolve around the y-axis. Setup, but **do not** evaluate.

$y^2 = 9 - x^2$

$x^2 = 9 - y^2$

$x = \pm\sqrt{9-y^2}$ $\quad x = \sqrt{9-y^2}$



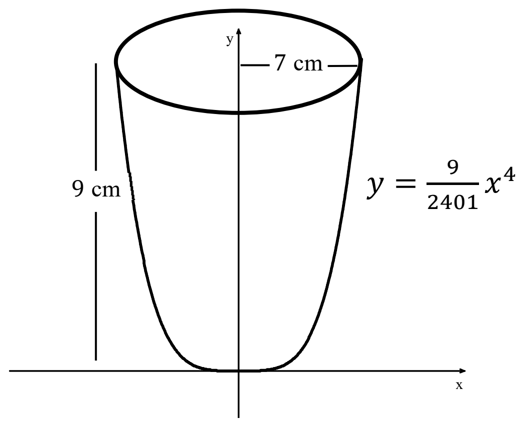
$$\boxed{\int_0^3 \pi(9-y^2)\, dy}$$

8.9 Disc Method: Revolve Around x or y Axis

Test Prep

11. **Calculator allowed, but show your steps!** Mr. Brust is pouring his favorite liquid gelatin into a mixing bowl. The bowl's shape can be obtained by revolving the curve $y = \frac{9}{2401}x^4$ from $x = 0$ to $x = 7$ about the y-axis, where x and y are measured in centimeters. The gelatin is poured into the empty bowl at a constant rate of 14 cubic centimeters per second.

Let h be the depth, in centimeters, of gelatin in the bowl. How fast was the depth of the gelatin in the bowl increasing when $h = 4$? Leave your answer in terms of π. Indicate all units of measure.



$$V = \int_0^h \pi\left(\left(\frac{2401}{9}y\right)^{\frac{1}{4}}\right)^2 dy$$

$$V = \int_0^h \pi\left(\frac{2401}{9}y\right)^{\frac{1}{2}} dy$$

$$V = \pi\int_0^h \left(\frac{49}{3}\sqrt{y}\right) dy$$

$$\frac{dV}{dt} = \frac{49}{3}\pi\sqrt{h}\frac{dh}{dt}$$

At $h = 4$,

$$14 = \frac{49}{3}\pi\sqrt{4}\frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{3}{7\pi}\ \textbf{centimeters per second.}}$$