Calculus

9.1 Parametric Equations

Notes

Write your questions and thoughts here!

We have been looking at graphs of one equation with two variables, typically x and y. Now we are looking at three variables that will represent a curve in the plane.

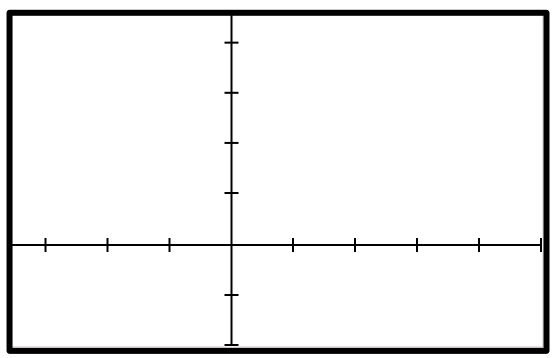
In the rectangular equation we are able to determine where the object is located at a point (x, y), but with the addition of the third variable (often t), we are able to determine **when** the object was at a point (x, y). NOTE: the third variable t is often time, but not always.

Parametric Equations

If f and g are continuous functions of t on an interval I, then the equations $x = f(t)$ and $y = g(t)$ are parametric equations and t is the parameter. You can sketch the curve of a parametric by substituting in values for t.

1. Sketch the curve with the following parametrization: $x(t) = 2t$ and $y(t) = t^2 - 1$, with $-1 \leq t \leq 2$.

t	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
x							
y							



```
WINDOW
 Tmin=-1
 Tmax=2
 Tstep=0.2
 Xmin=-3.5
 Xmax=5
 Xscl=1
 Ymin=-2
 Ymax=4.5
 Yscl=1
```

To find the rectangular equation when you are given the parametric equations, eliminate the parameter t through substitution.

2. Given $x(t) = 2t, y(t) = t^2 - 1$. Find the rectangular equation by eliminating the parameter.

3. Given the parametric equations $x(t) = 2\cos t$ and $y(t) = 2\sin t$. Eliminate the parameter.

Write your questions and thoughts here!

Derivative of a Parametric Equation

The derivative of a parametric given by $x = f(t)$ and $y = g(t)$ is found by the following:

4. Given $x(t) = t^{\frac{1}{2}}$ and $y(t) = \frac{1}{4}(t^2 - 4)$ for $t \geq 0$. Find $\frac{dy}{dx}$

5. Given $x(t) = e^{2t}$ and $y(t) = \cos t$ for $t \geq -1$. Find the equation of a tangent line when $t = \frac{\pi}{2}$.

9.1 Parametric Equations

Calculus

Practice

1. For the given parametric equations, eliminate the parameter and write the corresponding rectangular equation. $x = e^{-t}$ and $y = e^{2t} - 1$.	2. Let C be a curve described by the parametrization $x = 5t$ and $y = t^4 + 3$. Find an expression for the slope of the line tangent to C at any point (x, y).
3. The position of a particle at any time $t \geq 0$ is given by $x(t) = 3t^2 + 1$ and $y(t) = \frac{2}{3}t^3$. Find $\frac{dy}{dx}$ as a function of x.	4. A particle moves along the curve $xy + y = 9$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$?

5. A curve is described by the parametric equations $x = t\cos t$ and $y = t\sin t$. Find the equation of the line tangent to the curve at the point determined by $t = \pi$.

6. **Calculator active.** The coordinates $\left(x(t), y(t)\right)$ of the position of a drone change at rates given by $x'(t) = 2t^3$ and $y'(t) = t^{\frac{1}{2}}$, where $x(t)$ and $y(t)$ are measured in meters and t is measured in seconds. At what time t, for $0 \leq t \leq 2$, does the slope of the line tangent to its path have a slope of 1.5?

7. A curve in the xy-plane is defined by the parametric equations $x(t) = \cos(3t)$ and $y(t) = \sin(3t)$ for $t \geq 0$. What is the value of $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$?

8. A curve is defined by the parametric equations $x(t) = at^2 + b$ and $y(t) = ct - b$, where a, b, and c are nonzero constants. What is the slope of the line tangent to the curve at the point $(x(t), y(t))$ when $t = 2$?

9. **No Calculator.** For $0 \leq t \leq 11$ the parametric equations $x = 3\sin t$ and $y = 2\cos t$ describe the elliptical path of an object. At the point where $t = 11$, the object travels along a line tangent to the path at that point. What is the slope of that line?

10. A particle moves in the xy-plane so that its position for $t \geq 0$ is given by the parametric equations $x(t) = 2kt^2$ and $y(t) = 3t$, where k is a positive constant. When $t = 2$ the line tangent to the particle's path has a slope of 4. What is the value of k?

11. Find the equation of the line tangent to the curve defined parametrically by the equations $x(t) = t^3 + 2t$ and $y(t) = 2t^4 + 2t^2$ when $t = 1$.

12. For what values of t does the curve given by the parametric equations $x(t) = \frac{1}{4}t^4 - \frac{9}{2}t^2$ and $y(t) = 3t^3 + 2t$ have a vertical tangent?

13. Suppose a curve is given by the parametric equations $x = f(t)$ and $y = g(t)$, for all $t > 1$ and $\frac{dy}{dt} = \frac{t^2+2}{t-1} * \frac{dx}{dt}$. What is the value of $\frac{dy}{dx}$ when $t = 2$?

9.1 Parametric Equations

Test Prep

14. A curve is defined parametrically by $x(t) = t^2$ and $y(t) = t^3 - 3t$. Find the points on the graph where the tangent line is horizontal or vertical.

15. **Free Response**. Consider the curve given by the parametric equations $y = t^3 - 12t$ and $x = \frac{1}{2}t^2 - t$.

a. Find $\frac{dy}{dx}$ in terms of t.

b. Write an equation for the line tangent to the curve at the point where $t = -1$.

c. Find the x and y coordinates for each critical point on the curve and identify each point as having a vertical or horizontal tangent.

---

16. A curve is given by the parametric equations $x(t) = 5t^3 - 5$ and $y(t) = t^2 + 7$. What is the equation of the tangent line to the curve when $t = 1$?

A. $x = 0$ B. $y = \frac{2}{15}x + 8$ C. $y = \frac{2}{15}x + 1$ D. $y = 8$ E. $y = \frac{15}{2}x + 7$