9.4 Derivatives of Vector-Valued Functions

Calculus

Solutions

Practice

Each problem contains a vector-valued function. Find the given first or second derivative.

1. $f(t) = \langle 4t^3 + 2t^2 + 7t,\ 4t^2 + 3t \rangle$, then $f'(t) =$

$$\boxed{f'(t) = \langle 12t^2 + 4t + 7,\ 8t + 3 \rangle}$$

2. $f(t) = \langle 3\sin 2t, 4\cos 3t \rangle$, then $f'\left(\frac{\pi}{6}\right) =$

$$f'(t) = \langle 6\cos(2t),\ -12\sin(3t) \rangle$$

$$f'\left(\tfrac{\pi}{6}\right) = \langle 6\cos\left(\tfrac{\pi}{3}\right),\ -12\sin\left(\tfrac{\pi}{2}\right) \rangle$$

$$= \langle 6\left(\tfrac{1}{2}\right),\ -12(1) \rangle$$

$$\boxed{\langle 3, -12 \rangle}$$

3. $f(t) = \langle 3e^{2t},\ 5e^{4t} \rangle$, then $f''(t) =$

$$f'(t) = \langle 6e^{2t},\ 20e^{4t} \rangle$$

$$\boxed{f''(t) = \langle 12e^{2t},\ 80e^{4t} \rangle}$$

4. $f(t) = \langle t^{-2},\ (t+1)^{-1} \rangle$, then $f''(-2) =$

$$f'(t) = \langle -2t^{-3},\ -(t+1)^{-2} \rangle$$

$$f''(t) = \langle 6t^{-4},\ 2(t+1)^{-3} \rangle$$

$$f''(-2) = \langle \frac{6}{(-2)^4},\ \frac{2}{(-2+1)^{-3}} \rangle$$

$$\langle \frac{6}{16},\ \frac{2}{-1} \rangle$$

$$\boxed{\langle \tfrac{3}{8}, -2 \rangle}$$

5. $f(t) = \langle e^t + e^{-t},\ e^t - e^{-t} \rangle$, then $f'(t) =$

$$\boxed{f'(t) = \langle e^t - e^{-t},\ e^t + e^{-t} \rangle}$$

6. $f(t) = \langle 2\sin 4t, 2\cos 3t \rangle$, then $f'(t) =$

$$\boxed{f'(t) = \langle 8\cos(4t),\ -6\sin(3t) \rangle}$$

7. $f(t) = \langle t\sin t,\ t\cos t \rangle$, then $f'\left(\frac{\pi}{2}\right) =$

$$f'(t) = \langle \sin t + t\cos t,\ \cos t - t\sin t \rangle$$

$$f'\left(\tfrac{\pi}{2}\right) = \langle \sin\tfrac{\pi}{2} + \tfrac{\pi}{2}\cos\tfrac{\pi}{2},\ \cos\tfrac{\pi}{2} - \tfrac{\pi}{2}\sin\tfrac{\pi}{2} \rangle$$

$$= \langle 1 + \tfrac{\pi}{2}(0),\ 0 - \tfrac{\pi}{2}(1) \rangle$$

$$\boxed{\langle 1, -\tfrac{\pi}{2} \rangle}$$

8. $f(t) = \langle 3t^2 + 6t + 1,\ 4t^3 - 2t^2 + 6t \rangle$, then $f'(1) =$

$$f'(t) = \langle 6t + 6,\ 12t^2 - 4t + 6 \rangle$$

$$f'(1) = \langle 6 + 6,\ 12 - 4 + 6 \rangle$$

$$\boxed{f'(1) = \langle 12, 14 \rangle}$$

9. The path of a particle moving along a path in the xy-plane is given by the vector-valued function, $f(t) = \langle t^3 + 2t^2 + t,\ 2t^3 - 4t\rangle$. Find the slope of the path of the particle at $t = 3$.

$$f'(t) = \langle 3t^2 + 4t + 1,\ 6t^2 - 4 \rangle$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}\Bigg|_{t=3} = \frac{6(3)^2 - 4}{3(3)^2 + 4(3) + 1} = \frac{50}{40} = \boxed{5/4}$$

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10. The position of a particle moving in the xy-plane is defined by the vector-valued function, $f(t) = \langle t^3 - 6t^2,\ 2t^3 - 9t^2 - 24t\rangle$. For what value of $t \geq 0$ is the particle at rest?

$$x'(t) = 3t^2 - 12t$$
$$3t(t-4) = 0$$

$$y'(t) = 6t^2 - 18t - 24$$
$$6(t^2 - 3t - 4) = 0$$
$$6(t-4)(t+1) = 0$$

$$\boxed{t = 4}$$

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Test Prep

11. **Calculator active.** The path of a particle moving along a path in the xy-plane is given by the vector-valued function f and f' is defined by $f'(t) = \langle t^{-1},\ 2ke^{kt}\rangle$ where k is a positive constant. The line $y = 4x + 5$ is parallel to the line tangent to the path of the particle at the point where $t = 2$. What is the value of k?

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$$\frac{dy}{dx} = \frac{2ke^{kt}}{t^{-1}}\Bigg|_{t=4} = \frac{2ke^{2k}}{2^{-1}} \rightarrow 4ke^{2k} = 4$$

$$ke^{2k} = 1$$

$$\boxed{k \simeq 0.426}$$

12. At time t, $0 \leq t \leq 2\pi$, the position of a particle moving along a path in the xy-plane is given by the vector-valued function, $f(t) = \langle t\sin t, \cos 2t\rangle$. Find the slope of the path of the particle at time $t = \frac{\pi}{2}$.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2\sin(2t)}{\sin t + t\cos t}\Bigg|_{t=\pi/2} = \frac{-2\sin(\pi)}{\sin\frac{\pi}{2} + \frac{\pi}{2}\cos(\frac{\pi}{2})} = \boxed{0}$$