9.5 Integrating Vector-Valued Functions   Notes

Integration of Vector-Valued Functions
If $r(t) = \langle f(t), g(t) \rangle$ then

1. Find $r(t)$ if $r'(t) = \langle 4e^{2t}, 2e^t \rangle$ and $r(0) = \langle 2, 0 \rangle$

2. Find $r(t)$ if $r'(t) = \langle \sec^2 t, \frac{1}{1+t^2} \rangle$

3. $\int_{-1}^{1} \langle t^3, t^{\frac{1}{5}} \rangle \, dt$

9.5 Integrating Vector-Valued Functions
Calculus   Practice

For problems 1-6, find the vector-valued function $f(t)$ that satisfies the given initial conditions.

1. $f(0) = \langle 2, 4 \rangle$, $f'(t) = \langle 2e^t, 3e^{3t} \rangle$

2. $f(0) = \langle \frac{1}{2}, -1 \rangle$, $f'(t) = \langle te^{-t^2}, -e^{-t} \rangle$

3. $f(0) = \langle 3, 1 \rangle$, $f'(t) = \langle 6t^2, 4t \rangle$

4. $f(0) = \langle -2, 5 \rangle$, $f'(t) = \langle 2 \cos t, -3 \sin t \rangle$

5. $f'(0) = \langle 3, 0 \rangle$, $f(0) = \langle 0, 3 \rangle$,
   $f''(t) = \langle 5 \cos t, -2 \sin t \rangle$

6. $f'(0) = \langle 0, 2 \rangle$, $f(0) = \langle 3, 0 \rangle$, $f''(t) = \langle 4t^3, 3t^2 \rangle$

7. **Calculator active.** For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 1$, the particle is at position $(2, 4)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+3}}{e^t}$ and $\frac{dy}{dt} = \cos^2 t$. Find the x-coordinate of the particles position at time $t = 5$.

8. The instantaneous rate of change of the vector-valued function $f(t)$ is given by $f'(t) = \langle 8t^3 + 2t, 10t^4 \rangle$. If $f(1) = \langle 3, 7 \rangle$, what is $f(-1)$?

9. **Calculator active.** At time $t \geq 0$, a particle moving in the xy-plane has velocity vector given by $v(t) = \langle 3t^2, 3 \rangle$. If the particle is at point $(1, 2)$ at time $t = 0$, how far is the particle from the origin at time $t = 2$?

10. **Calculator active.** At time $t \geq 0$, a particle moving in the xy-plane has velocity vector given by $v(t) = \langle 2, \frac{\cos t}{e^t} \rangle$. If the particle is at point $(1, 2)$ at time $t = 0$, how far is the particle from the origin at time $t = 3$?

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11. **Calculator active.** A remote controlled car travels on a flat surface. The car starts at the point with coordinates $(7, 6)$ at time $t = 0$. The coordinates $(x(t), y(t))$ of the position change at rates given by $x'(t) = -10 \sin t^2$ and $y'(t) = 9 \cos(2 + \sqrt{t})$, where $x(t)$ and $y(t)$ are measured in feet and t is measured in minutes. Find the y-coordinate of the position of the car at time $t = 1$.

12. The instantaneous rate of change of the vector-valued function $f(t)$ is given by $f'(t) = \langle 2 + 20t - 4t^3, \ 6t^2 + 2t \rangle$. If $f(1) = \langle 5, -3 \rangle$, what is $f(-1)$?