9.6 Motion using Parametric and Vector-Valued Functions

Calculus

Solutions **Practice**

For each problem, a particle moves in the xy-plane where the coordinates are defined at any time t by the position function given in parametric or vector form.

1. $x(t) = 4t^2$ and $y(t) = 2t - 1$. Find the velocity vector at time $t = 1$.

$x'(t) = 8t \qquad y'(t) = 2$

$x'(1) = 8 \qquad y'(1) = 2$

$\boxed{v'(1) = \langle 8, 2 \rangle}$

2. $x(t) = e^{-t}$ and $y(t) = e^{t}$. Find the acceleration vector at time $t = 1$.

$x'(t) = -e^{-t} \qquad y'(t) = e^{t}$

$x''(t) = e^{-t} \qquad y''(t) = e^{t}$

$x''(1) = e^{-1} \qquad y''(1) = e^{1}$

$\boxed{a(t) = \langle \frac{1}{e}, e \rangle}$

3. $(x(t), y(t)) = (6 - 2t, t^2 + 3)$. In which direction is the particle moving as it passes through the point $(4, 4)$?

$x = 4$

$4 = 6 - 2t$

$-2 = -2t$

$1 = t$

$y = 4$

$4 = t^2 + 3$

$1 = t^2$

$\pm 1 = t$

$v'(t) = \langle -2, 2t \rangle$

$v'(1) = \langle -2, 2 \rangle$

neg pos

$\boxed{\text{Particle is moving left and up.}}$

4. A position vector is $r(t) = \langle \frac{2}{t},\ e^{4t} \rangle$ for time $t > 0$. What is the velocity vector at time $t = 1$?

$r'(t) = \langle -2t^{-2}, 4e^{4t} \rangle$

$\boxed{r'(1) = \langle -2, 4e^4 \rangle}$

5. $r(t) = \langle \ln(t^2 + 1), 3t^2 \rangle$ for $t > 0$. Find the velocity vector at time $t = 2$.

$v(t) = \langle \frac{2t}{t^2 + 1}, 6t \rangle$

$\boxed{v(2) = \langle \frac{4}{5}, 12 \rangle}$

6. $x(t) = 2\sin\frac{t}{2}$ and $y(t) = 2\cos\frac{t}{2}$ for time $t > 0$. Find the speed of the particle.

$x'(t) = \cos\left(\frac{t}{2}\right)$ $y'(t) = -\sin\left(\frac{t}{2}\right)$

speed $= \sqrt{\cos^2\left(\frac{t}{2}\right) + \sin^2\left(\frac{t}{2}\right)}$

Pythagorean Identity!

$\sqrt{1}$

$\boxed{1}$

7. **Calculator active.** $x(t) = t^2 + 1$ and $y(t) = \frac{4}{3}t^3$ for time $t \geq 0$. Find the total distance traveled from $t = 0$ to $t = 3$.

$x'(t) = 2t$ $y'(t) = 4t^2$

$\int_0^3 \sqrt{(2t)^2 + (4t^2)^2}\, dt$

$\boxed{37.3437}$

8. $p(t) = \langle \cos 2t, 2\sin t \rangle$. Find the velocity vector $v(t)$.

$\boxed{v(t) = \langle -2\sin 2t, 2\cos t \rangle}$

9. **Calculator active.** The velocity vector of a particle moving in the xy-plane has components given by $\frac{dx}{dt} = \cos t^2$ and $\frac{dy}{dt} = e^{t-2}$. At time $t = 3$, the position of the particle is $(1, 2)$. What is the y-coordinate of the position vector at time $t = 2$?

$2 + \int_3^2 e^{t-2}\,dt$

$\boxed{0.2817}$

10. At time $t \geq 0$, a particle moving in the xy-plane has velocity vector given by $v(t) = \langle t^3, 4t \rangle$. What is the acceleration vector when $t = 2$?

$a(t) = \langle 3t^2, 4 \rangle$

$\boxed{a(2) = \langle 12, 4 \rangle}$

11. The acceleration vector of a particle moving in the xy-plane is given by $a(t) = \langle 2, 3 \rangle$. When $t = 0$ the velocity vector is $\langle 3, 1 \rangle$ and the position vector is $\langle 1, 5 \rangle$. Find the position when time $t = 2$.

$V(t) = \langle 2t + C_1, 3t + C_2 \rangle$

$V(0) = \langle 3, 1 \rangle \rightarrow \langle C_1 = 3, C_2 = 1 \rangle$

$V(t) = \langle 2t + 3, 3t + 1 \rangle$

$P(t) = \langle t^2 + 3t + C_1, \frac{3}{2}t^2 + t + C_2 \rangle$

$P(0) = \langle 1, 5 \rangle \rightarrow \langle C_1 = 1, C_2 = 5 \rangle$

$P(2) = \langle 4 + 6 + 1, \frac{3}{2}(4) + 2 + 5 \rangle$

$\boxed{P(2) = \langle 11, 13 \rangle}$

12. A particle moves on the curve $y = 2x$ so that the x-component has velocity $x'(t) = 3t^2 + 1$ for $t \geq 0$. At time $t = 0$, the particle is at the point $(2, 4)$. At what point is the particle when $t = 1$? [This one is tricky!]

$x(t) = t^3 + t + C$

$2 = 0 + 0 + C$

$2 = C$

$x(t) = t^3 + t + 2$

$y = 2x$

Position vector: $\langle t^3 + t + 2, 2x \rangle$

$2 \cdot x(t)$

$\langle t^3 + t + 2, 2t^3 + 2t + 4 \rangle$

$\boxed{\text{at } t = 1, (4, 8)}$

For problems 13-15: At time t, $0 \leq t \leq 2\pi$, the position of a particle moving along a path in the xy-plane is given by parametric equations $x(t) = \cos 2t$ and $y(t) = \sin 2t$.

13. Find the speed of the particle when $t = 1$.

$x'(t) = -2\sin(2t) \qquad y'(t) = 2\cos(2t)$

$\sqrt{4\sin^2(2t) + 4\cos^2(2t)}$

$\sqrt{4[\sin^2(2t) + \cos^2(2t)]}$

$\sqrt{4 \cdot [1]} = \boxed{2}$

14. Find the acceleration vector at time $t = \frac{\pi}{4}$.

$x''(t) = -4\cos(2t) \qquad y''(t) = -4\sin(2t)$

$x''(\frac{\pi}{4}) = -4\cos(\frac{\pi}{2}) \qquad y''(\frac{\pi}{4}) = -4\sin(\frac{\pi}{2})$

$= -4(0) \qquad = -4(1)$

$a(\frac{\pi}{4}) = \boxed{\langle 0, -4 \rangle}$

15. Find the distance traveled from $t = 0$ to $t = 3$.

$\int_0^3 (\text{speed})\,dt$

$\int_0^3 (2)\,dt \rightarrow 2t \Big|_0^3$

$6 - 0 = \boxed{6}$

16. **Calculator active.** A remote-controlled car moves along a flat surface over the time interval $0 \le t \le 30$ seconds. The position of the remote-controlled car at time t is given by the parametric equations $x(t) = 2t + \sin t$ and $y(t) = 2\cos(t - \sin t)$, where $x(t)$ and $y(t)$ are measured in feet. The derivatives of these functions are given by $x'(t) = 2 + \cos t$ and $y'(t) = -2\sin(t - \sin t)(1 - \cos t)$.

a. Write the equation for the line tangent to the path of the remote-controlled car at time $t = 3$ seconds.

$y - y_1 = m(x - x_1)$

$y - y(3) = \left.\frac{dy}{dx}\right|_{t=3} (x - x(3))$

$y(3) \approx -1.9206 \quad x(3) \approx 6.141$

$\left.\frac{dy}{dx}\right|_{t=3} = \frac{-2\sin(3 - \sin 3)(1 - \cos 3)}{2 + \cos 3}$

$= -1.099$

$\boxed{y + 1.921 = -1.099(x - 6.141)}$

b. Find the speed of the remote-controlled car at time $t = 15$ seconds.

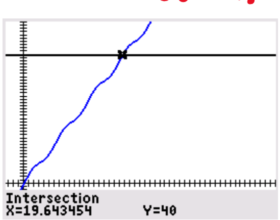
$\text{Speed} = \sqrt{[x'(15)]^2 + [y'(15)]^2}$

$\boxed{3.6569 \text{ feet/second}}$

c. Find the acceleration vector of the remote-controlled car at the time when the car is at the point with x-coordinate 40.

$40 = 2t + \sin t$

Graph and find point of intersection.



$t \approx 19.643454 \rightarrow A$

store

$x''(A) \approx -0.713$

use math 8 on $x'(t)$ and $y'(t)$

$y''(A) \approx -0.293$

$\boxed{\langle -0.713, -0.293 \rangle}$