Calculus

Write your questions and thoughts here!

9.7 Differentiating in Polar Form

Notes

(x, y) is for a **rectangular** coordinate system.

(r, θ) is for a **polar** coordinate system.

r is a directed distance from the origin to a point P.

θ is the directed angle

Polar ⟹ Rectangular	**Rectangular ⟹ Polar**
$x = r\cos\theta$	$\tan\theta = \frac{y}{x}$
$y = r\sin\theta$	$r^2 = x^2 + y^2$

Convert the following from polar form to rectangular form.

1. $r\cos\theta = -4$
2. $4r\cos\theta = r^2$
3. $\frac{4}{2\cos\theta - \sin\theta} = r$

Slope of a Curve in Polar Form

A curve in polar form is given by $r = f(\theta)$, then its rectangular coordinates are given by $\begin{cases} x = f(\theta)\cos\theta \\ y = f(\theta)\sin\theta \end{cases}$. The derivative $\frac{dy}{dx}$ is defined the same way as the derivative of a parametric equation.

$$\frac{dy}{dx} =$$

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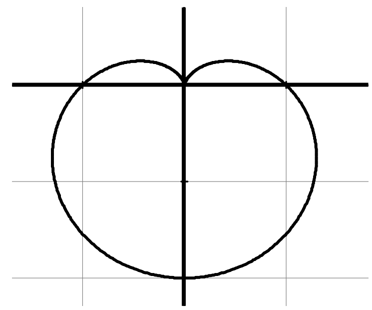
The following is an example of a common problem found on the AP Exam!

4. What is the slope of the line tangent to the polar curve $r = 1 + 2\sin\theta$ at $\theta = 0$?

5. Find the value(s) of θ where the polar graph $r = 1 - \sin\theta$ on the interval $0 \le \theta \le 2\pi$ has horizontal and vertical tangent lines.



9.7 Differentiating in Polar Form

Calculus

Practice

Problems 1-5 are pre-calculus review on polar form.

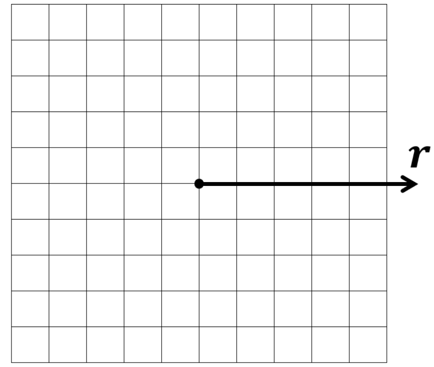
1. Find the corresponding rectangular coordinates for the polar coordinates $\left(7, \frac{5\pi}{4}\right)$.

2. **Calculator active.** Find two sets of polar coordinates for the rectangular coordinate $(4, -2)$. Limit your answers on the interval $0 \le \theta \le 2\pi$.

3. Convert the rectangular equation $x^2 + y^2 = 16$ to a polar equation.

4. Convert the polar equation $r = 3\sec\theta$ to a rectangular equation.

5. Sketch the polar curve $r = 2\cos 3\theta$ for $0 \leq \theta \leq \pi$ **without** a calculator, then check your answer.



Find the slope of the line tangent to the polar curve at the given value of $\boldsymbol{\theta}$.

6. $r = 3\theta$ at $\theta = \frac{\pi}{2}$.

7. $r = \frac{5}{3-\cos\theta}$ at $\theta = \frac{3\pi}{2}$.

8. $r = \cos\theta$ at $\theta = \frac{\pi}{3}$.

9. $r = 2(1 - \sin\theta)$ at $\theta = 0$.

10. A particle moves along the polar curve $r = 3\cos\theta$ so that $\frac{d\theta}{dt} = 2$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$. *Hint: remember implicit differentiation?*

11. A polar curve is given by the equation $r = \frac{15\theta}{\theta^2+1}$ for $\theta \geq 0$. What is the instantaneous rate of change of r with respect to θ when $\theta = 1$?

12. Find the value(s) of θ where the polar graph $r = 2 - 2\cos\theta$ has a horizontal tangent line on the interval $0 \leq \theta \leq 2\pi$. Use a graphing calculator to verify your answers.

13. Find the value(s) of θ where the polar graph $r = 3 - 3\sin\theta$ has a vertical tangent line on the interval $0 \leq \theta \leq 2\pi$. Use a graphing calculator to verify your answers.

14. **Calculator active.** For a certain polar curve $r = f(\theta)$, it is known that $\frac{dx}{d\theta} = \cos\theta - \theta\sin\theta$ and $\frac{dy}{d\theta} = \sin\theta + \theta\cos\theta$. What is the value of $\frac{d^2y}{dx^2}$ at $\theta = 3$?

9.7 Differentiating in Polar Form

15. A polar curve is given by the differentiable function $r = f(\theta)$ for $0 \leq \theta \leq 2\pi$. If the line tangent to the polar curve at $\theta = \frac{\pi}{6}$ is vertical, which of the following must be true?

A. $f\left(\frac{\pi}{6}\right) = 0$ B. $f'\left(\frac{\pi}{6}\right) = 0$ C. $\frac{1}{2}f\left(\frac{\pi}{6}\right) - \frac{\sqrt{3}}{2}f'\left(\frac{\pi}{6}\right) = 0$ D. $\frac{\sqrt{3}}{2}f'\left(\frac{\pi}{6}\right) - \frac{1}{2}f\left(\frac{\pi}{6}\right) = 0$

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16. **Calculator active.** For $0 \leq t \leq 8$, a particle moving in the xy-plane has position vector $\langle x(t),\ y(t)\rangle = \langle \sin(2t), t^2 - t\rangle$, where $x(t)$ and $y(t)$ are measured in meters and t is measured in seconds.

a. Find the speed of the particle at time $t = 3$ seconds. Indicate units of measure.

b. At time $t = 5$ seconds, is the speed of the particle increasing or decreasing? Explain your answer.

c. Find the total distance the particle travels over the time interval $0 \leq t \leq 6$ seconds.

d. At time $t = 8$ seconds, the particle begins moving in a straight line. For $t \geq 8$, the particle travels with the same velocity vector that it had at time $t = 8$ seconds. Find the position of the particle at time $t = 11$ seconds.