9.9 Area Bounded by Two Polar Curves

Calculus

Solutions

Practice

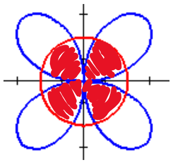
1. Find the area of the common interior of the polar graphs $r = 4\sin 2\theta$ and $r = 2$.

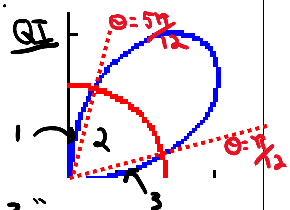
$2=4\sin 2\theta$

$\frac{1}{2}=\sin 2\theta$

$2\theta=\frac{\pi}{6},\frac{5\pi}{6}$

$\theta=\frac{\pi}{12},\frac{5\pi}{12}$





3 "areas"

$$\frac{1}{4}A=\frac{1}{2}\int_0^{\pi/12}[4\sin 2\theta]^2 d\theta+\frac{1}{2}\int_{\pi/12}^{5\pi/12}[2]^2 d\theta+\frac{1}{2}\int_{5\pi/12}^{\pi/2}[4\sin 2\theta]^2 d\theta$$

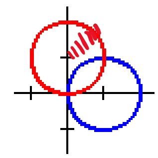
$\frac{1}{4}A\approx 2.4567$

$$\boxed{A\approx 9.8268}$$

2. Find the area of the common interior of the polar graphs $r = 2\cos\theta$ and $r = 2\sin\theta$.

$2\cos\theta=2\sin\theta$

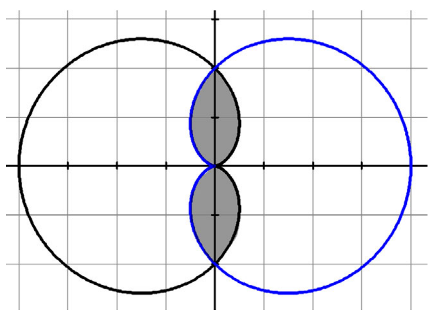
$\theta=\frac{\pi}{4}$



$$\frac{1}{2}\int_0^{\pi/4}[2\sin\theta]^2 d\theta+\frac{1}{2}\int_{\pi/4}^{\pi/2}[2\cos\theta]^2 d\theta$$

$$\boxed{0.5708 \text{ or } \frac{\pi}{2}-1}$$

3. The polar curves $r = 2 - 2\cos\theta$ and $r = 2 + 2\cos\theta$ are shown below.



Which of the following gives the total area of the shaded regions? QI $\frac{1}{2}\int_0^{\pi/2}[2-2\cos\theta]^2 * 4 =$

A. $\int_0^{\pi}(2+2\cos\theta)^2\ d\theta$

B. $\int_{\pi/2}^{\pi}(2+2\cos\theta)^2\ d\theta$

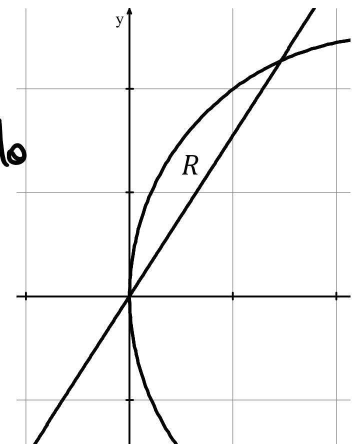
C. $8\int_0^{\frac{\pi}{2}}(1-\cos\theta)^2\ d\theta$

D. $\int_0^{\frac{\pi}{2}}((2-2\cos\theta)^2+(2+2\cos\theta)^2)\, d\theta$

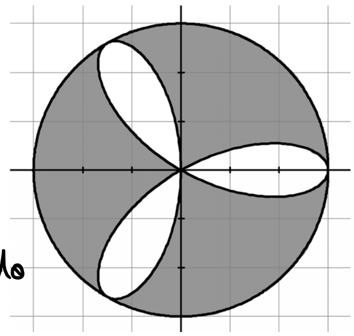
4. Let R be the region in the first quadrant that is bounded above by the polar curve $r = 5\cos\theta$ and below by the line $\theta = 1$, as shown in the figure below. What is the area of R?

$$\frac{1}{2}\int_1^{\pi/2}[5\cos\theta]^2 d\theta$$

$$\boxed{0.7259}$$



5. The figure below shows the graphs of the polar curves $r = 3\cos 3\theta$ and $r = 3$. What is the sum of the areas of the shaded regions?



Circle $= \pi(3)^2$

$= 9\pi$

One petal $=$

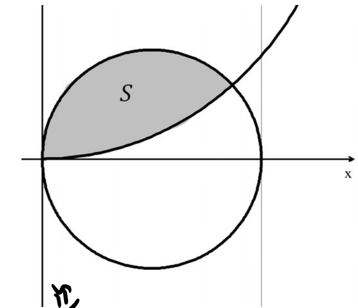
$\frac{1}{2}\int_{-\pi/6}^{\pi/6} [3\cos(3\theta)]^2 d\theta$

Circle $-$ 3 petals

$9\pi - \frac{3}{2}\int_{-\pi/6}^{\pi/6} [3\cos(3\theta)]^2 d\theta$

$\boxed{21.2057}$

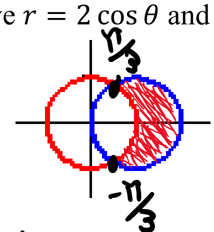
6. Let S be the region in the 1st Quadrant bounded above by the graph of the polar curve $r = \cos\theta$ and bounded below by the graph of the polar curve $r = \frac{5}{2}\theta$, as shown in the figure above. The two curves intersect when $\theta = 0.373$. What is the area of S?



$\frac{1}{2}\int_0^{0.373} [\frac{5}{2}\theta]^2 d\theta + \frac{1}{2}\int_{0.373}^{\pi/2} [\cos\theta]^2 d\theta$

$\boxed{0.2686}$

7. Find the area inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = 1$.



$2\cos\theta = 1$

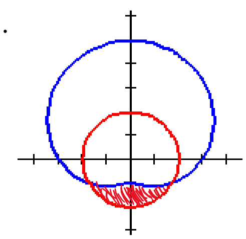
$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

$\frac{1}{2}\int_{-\pi/3}^{\pi/3} [2\cos\theta]^2 d\theta - \frac{1}{2}\int_{-\pi/3}^{\pi/3} [1]^2 d\theta$

$\boxed{1.913}$

8. Write an integral expression that represents the area of the region outside the polar curve $r = 3 + 2\sin\theta$ and inside the polar curve $r = 2$.



$3 + 2\sin\theta = 2$

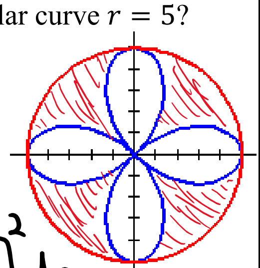
$2\sin\theta = -1$

$\sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\boxed{\frac{1}{2}\int_{7\pi/6}^{11\pi/6} \left[2^2 - [3 + 2\sin\theta]^2\right] d\theta}$

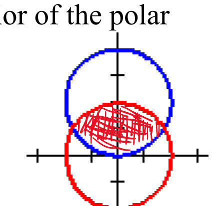
9. What is the total area outside the polar curve $r = 5\cos 2\theta$ and inside the polar curve $r = 5$?



$\pi(5)^2 - \frac{1}{2}\int_0^{2\pi} [5\cos(2\theta)]^2 d\theta$

$\boxed{39.270}$

10. Find the area of the common interior of the polar curves $r = 4\sin\theta$ and $r = 2$.



$4\sin\theta = 2$

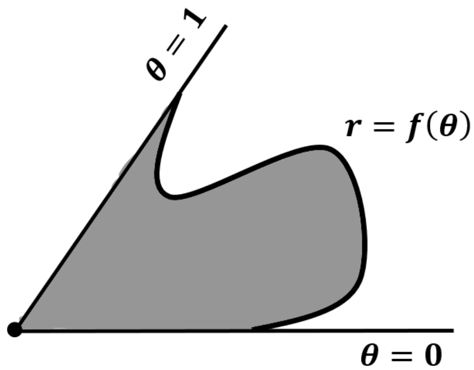
$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\frac{1}{2}\int_0^{\pi/6} [4\sin\theta]^2 d\theta + \frac{1}{2}\int_{\pi/6}^{5\pi/6} 2^2 d\theta + \frac{1}{2}\int_{5\pi/6}^{\pi} [4\sin\theta]^2 d\theta$

$\boxed{4.913}$

11.



θ	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
r	1	3	5	4	2

No calculator! Let R be the region bounded by the graph of the polar curve $r = f(\theta)$ and the lines $\theta = 0$ and $\theta = 1$, as shaded in the figure above. The table above gives values of the polar function $r = f(\theta)$ at selected values of θ. What is the approximation for the area of region R using a right Riemann sum with the four subintervals indicated by the data in the table?

Area of each Sector $= \frac{1}{2}\theta r^2$

$$\underbrace{\frac{1}{2}\cdot\frac{1}{4}\cdot 3^2}+\underbrace{\frac{1}{2}\cdot\frac{1}{4}\cdot 5^2}+\underbrace{\frac{1}{2}\cdot\frac{1}{4}\cdot 4^2}+\underbrace{\frac{1}{2}\cdot\frac{1}{4}\cdot 2^2}$$

$$\frac{1}{8}\cdot 9 + \frac{1}{8}\cdot 25 + \frac{1}{8}\cdot 16 + \frac{1}{8}\cdot 4$$

$$\frac{54}{8}$$

$$\boxed{\frac{27}{4}}$$