6.14 Selecting Techniques for Antidifferentiation

Calculus

Solutions

Practice

Find the indefinite integral.

1. $\int (3\csc x \cot x - 1)\,dx$

$$\boxed{-3\csc x - x + C}$$

2. $\int 3x\left(\sqrt{x} - x^2\right)dx$

$$\int 3x^{\frac{3}{2}} - 3x^3\,dx$$

$$\frac{3x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{3x^4}{4} + C$$

$$\boxed{\frac{6}{5}x^{\frac{5}{2}} - \frac{3}{4}x^4 + C}$$

3. $\int \frac{1}{\sqrt{-x^2-10x-24}}\,dx$

$$-(x^2+10x+25) - 24 + 25$$

$$\int \frac{1}{\sqrt{1-(x+5)^2}}\,dx$$

$$\boxed{\sin^{-1}(x+5) + C}$$

4. $\int 2^x\,dx$

$$\boxed{\frac{1}{\ln 2}2^x + C}$$

5. $\int \frac{1}{x^2-4x+5}\,dx$

$(x^2-4x+4)+5-4$

$$\int \frac{1}{(x-2)^2+1}\,dx$$

$$\boxed{\tan^{-1}(x-2)+C}$$

6. $\int (5-\sec^2 x)\,dx$

$$\boxed{5x-\tan x + C}$$

7. $\int \sqrt{x}\left(x-\frac{3}{x}\right)dx$

$$\int x^{3/2}-3x^{-1/2}\,dx$$

$$\frac{x^{5/2}}{5/2}-\frac{3x^{1/2}}{1/2}+C$$

$$\boxed{\frac{2}{5}x^{5/2}-6x^{1/2}+C}$$

8. $\int \frac{1}{\sqrt{1-9x^2}}\,dx$

$u=3x$

$\frac{du}{3}=dx$

$$\int \frac{1}{\sqrt{1-u^2}}\frac{du}{3}$$

$$\boxed{\frac{1}{3}\sin^{-1}(3x)+C}$$

9. $\int \sec(5x)\tan(5x)\,dx$

$u=5x$

$\frac{du}{5}=dx$

$$\frac{1}{5}\int \sec u\tan u\,du$$

$$\boxed{\frac{1}{5}\sec(5x)+C}$$

10. $\int \frac{4x^2}{x-2}\,dx$

2	4	0	0
		8	16
	4	8	16

$$\int 4x+8+\frac{16}{x-2}\,dx$$

$$\boxed{2x^2+8x+16\ln|x-2|+C}$$

11. $\int \left(\frac{8}{x}-\frac{1}{x^2}+e^x\right)dx$

$$\boxed{8\ln|x|+\frac{1}{x}+e^x+C}$$

12. $\int \frac{\sin x}{1+\cos^2 x}\,dx$

$u=\cos x$

$du=-\sin x\,dx$

$\frac{du}{-\sin x}=dx$

$$\int \frac{\sin x}{1+u^2}\,\frac{du}{-\sin x}$$

$$\int \frac{1}{1+u^2}\,du$$

$$\boxed{-\tan^{-1}(\cos x)+C \quad \text{or} \quad \cot^{-1}(\cos x)+C}$$

13. $\int \frac{10x^2-24x+12}{5x-2}\,dx$

$$\begin{array}{r|l} & 2x-4+\frac{4}{5x-2} \\ 5x-2 & 10x^2-24x+12 \\ & -(10x^2-4x) \\ & \quad -20x+12 \\ & \quad -(-20x+8) \\ & \qquad 4 \end{array}$$

$$\int 2x-4+\frac{4}{5x-2}\,dx$$

$$\boxed{x^2-4x+\frac{4}{5}\ln|5x-2|+C}$$

Evaluate the definite integral.

14. $\int_1^4 \left(\frac{1}{\sqrt{x}} - x^2\right) dx$

$2x^{1/2} - \frac{x^3}{3} \Big|_1^4$

$\left[2\sqrt{4} - \frac{4^3}{3}\right] - \left[2 - \frac{1}{3}\right]$

$\left[4 - \frac{64}{3}\right] - \left[\frac{5}{3}\right]$

$4 - \frac{69}{3}$

$\boxed{-19}$

15. $\int_1^2 \left(3x^2 - \frac{4}{x^2} + 1\right) dx$

$x^3 + \frac{4}{x} + x \Big|_1^2$

$[8 + 2 + 2] - [1 + 4 + 1]$

$12 - 6$

$\boxed{6}$

16. $\int_0^{\pi} (\sin x - 1)\, dx$

$-\cos x - x \Big|_0^{\pi}$

$[-(-1) - \pi] - [-(1) - 0]$

$1 - \pi + 1$

$\boxed{2 - \pi}$

17. $\int_4^{16} -\sqrt{x}\, dx$

$-\frac{x^{3/2}}{3/2} = -\frac{2}{3} x^{3/2} \Big|_4^{16}$

$-\frac{2}{3}\left[4^3 - 2^3\right]$

$-\frac{2}{3}[56]$

$\boxed{-\frac{112}{3}}$

18. $\int_1^2 e^{1-x}\, dx$

$u = 1 - x$

$du = -dx$

$-du = dx$

$\int_0^{-1} e^u (-du)$

$-[e^u] \Big|_0^{-1}$

$-\left[\frac{1}{e} - 1\right]$

$\boxed{1 - \frac{1}{e}}$

19. $\int_0^1 \frac{2x}{\sqrt{x^2+1}}\, dx$

$u = x^2 + 1$

$\frac{du}{2x} = dx$

$\int_1^2 \frac{1}{\sqrt{u}}\, du$

$2\sqrt{u} \Big|_1^2$

$\boxed{2\sqrt{2} - 2}$

20. $\int_0^{\frac{\pi}{8}} \tan(2x)\sec^2(2x)\, dx$

$u = \tan(2x)$

$du = 2\sec^2(2x)\, dx$

$\frac{du}{2\sec^2(2x)} = dx$

$\frac{1}{2}\int_0^1 u\, du$

$\frac{1}{2}\left[\frac{u^2}{2}\right] \Big|_0^1$

$\frac{1}{4}[1 - 0] = \boxed{\frac{1}{4}}$

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Test Prep

21. $\int_{-1}^{1} \frac{2}{1+x^2}\,dx =$

$2\tan^{-1}(x) \Big|_{-1}^{1}$

$2\left[\pi/4 - (-\pi/4)\right]$

$2\left[2\pi/4\right]$

(A) $-\pi$ (B) $-\frac{\pi}{2}$ (C) 0 (D) $\frac{\pi}{2}$ (E) π

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22. $\int x\sqrt{3x}\,dx =$

$x = \sqrt{x^2}$

$\int \sqrt{3x^3}\,dx$

$\sqrt{3}\int x^{3/2}\,dx = \sqrt{3}\,\frac{x^{5/2}}{5/2} + C$

(A) $\frac{2\sqrt{3}}{5}x^{\frac{5}{2}} + C$ (B) $\frac{5\sqrt{3}}{2}x^{\frac{5}{2}} + C$ (C) $\frac{\sqrt{3}}{2}x^{\frac{1}{2}} + C$ (D) $2\sqrt{3x} + C$ (E) $\frac{5\sqrt{3}}{2}x^{\frac{3}{2}} + C$