6.14 Selecting Techniques for Antidifferentiation

Calculus

Solutions

Practice

Find the indefinite integral.

1. $\int (3\csc x \cot x - 1)\,dx$

$$\boxed{-3\csc x - x + C}$$

2. $\int 3x\left(\sqrt{x} - x^2\right)dx$

$$\int 3x^{\frac{3}{2}} - 3x^3\,dx$$

$$\frac{3x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{3x^4}{4} + C$$

$$\boxed{\frac{6}{5}x^{\frac{5}{2}} - \frac{3}{4}x^4 + C}$$

3. $\int \frac{1}{\sqrt{-x^2-10x-24}}\,dx$

$$-(x^2+10x+25)-24+25$$

$$\int \frac{1}{\sqrt{1-(x+5)^2}}\,dx$$

$$\boxed{\sin^{-1}(x+5) + C}$$

4. $\int 2^x \, dx$

$$\boxed{\frac{1}{\ln 2} 2^x + C}$$

5. $\int \frac{1}{x^2-4x+5} dx$

$(x^2-4x+4)+5-4$

$$\int \frac{1}{(x-2)^2+1} dx$$

$$\boxed{\tan^{-1}(x-2) + C}$$

6. $\int (5-\sec^2 x) \, dx$

$$\boxed{5x - \tan x + C}$$

7. $\int \sqrt{x}\left(x-\frac{3}{x}\right) dx$

$$\int x^{3/2} - 3x^{-1/2} \, dx$$

$$\frac{x^{5/2}}{\cancel{5/2}} - \frac{3x^{1/2}}{\cancel{1/2}} + C$$

$$\boxed{\frac{2}{5} x^{5/2} - 6x^{1/2} + C}$$

8. $\int \frac{1}{\sqrt{1-9x^2}} dx$

$u = 3x$

$\frac{du}{3} = dx$

$$\int \frac{1}{\sqrt{1-u^2}} \frac{du}{3}$$

$$\boxed{\frac{1}{3}\sin^{-1}(3x) + C}$$

9. $\int \sec(5x)\tan(5x) \, dx$

$u = 5x$

$\frac{du}{5} = dx$

$$\frac{1}{5}\int \sec u \tan u \, du$$

$$\boxed{\frac{1}{5}\sec(5x) + C}$$

10. $\int \frac{4x^2}{x-2} dx$

$$\begin{array}{r|rrr} 2 & 4 & 0 & 0 \\ & & 8 & 16 \\ \hline & 4 & 8 & 16 \end{array}$$

$$\int 4x + 8 + \frac{16}{x-2} \, dx$$

$$\boxed{2x^2 + 8x + 16\ln|x-2| + C}$$

11. $\int_{-\infty}^{0} e^{3x} \, dx$

$u = 3x$

$\frac{du}{3} = dx$

$$\lim_{t \to -\infty} \int_t^0 e^{3x} \, dx$$

$$\lim_{t \to -\infty} \int_{3t}^0 e^u \frac{du}{3}$$

$$\lim_{t \to -\infty} \frac{e^u}{3} \Big|_{3t}^0$$

$$\lim_{t \to -\infty} \left[\frac{e^0}{3} - \frac{e^{3t}}{3}\right] = \frac{1}{3} - \frac{1}{3e^{\infty}} = \boxed{\frac{1}{3}}$$

12. $\int \frac{\sin x}{1+\cos^2 x} dx$

$u = \cos x$

$du = -\sin x \, dx$

$\frac{du}{-\sin x} = dx$

$$\int \frac{\sin x}{1+u^2} \frac{du}{-\sin x}$$

$$\int \frac{1}{1+u^2} du$$

$$\boxed{-\tan^{-1}(\cos x) + C \quad \text{or} \quad \cot^{-1}(\cos x) + C}$$

13. $\int \frac{10x^2-24x+12}{5x-2} dx$

$$\begin{array}{r|l} & 2x - 4 + \frac{4}{5x-2} \\ \hline 5x-2 & 10x^2 - 24x + 12 \\ & -(10x^2 - 4x) \\ \hline & \quad -20x + 12 \\ & \quad -(-20x + 8) \\ \hline & \qquad 4 \end{array}$$

$$\int 2x - 4 + \frac{4}{5x-2} \, dx$$

$$\boxed{x^2 - 4x + \frac{4}{5}\ln|5x-2| + C}$$

Evaluate the definite integral.

14. $\int_1^4 \left(\frac{1}{\sqrt{x}} - x^2\right) dx$

$2x^{1/2} - \frac{x^3}{3} \Big|_1^4$

$\left[2\sqrt{4} - \frac{4^3}{3}\right] - \left[2 - \frac{1}{3}\right]$

$\left[4 - \frac{64}{3}\right] - \left[\frac{5}{3}\right]$

$4 - \frac{69}{3}$

$\boxed{-19}$

15. $\int_1^2 \left(3x^2 - \frac{4}{x^2} + 1\right) dx$

$x^3 + \frac{4}{x} + x \Big|_1^2$

$[8+2+2] - [1+4+1]$

$12 - 6$

$\boxed{6}$

16. $\int_0^\pi (\sin x - 1)\, dx$

$-\cos x - x \Big|_0^\pi$

$[-(-1) - \pi] - [-(1) - 0]$

$1 - \pi + 1$

$\boxed{2 - \pi}$

17. $\int_4^{16} -\sqrt{x}\, dx$

$-\frac{x^{3/2}}{3/2} = -\frac{2}{3} x^{3/2} \Big|_4^{16}$

$-\frac{2}{3}\left[4^3 - 2^3\right]$

$-\frac{2}{3}[56]$

$\boxed{-\frac{112}{3}}$

18. $\int_1^2 e^{1-x}\, dx$

$u = 1-x$

$du = -dx$

$-du = dx$

$\int_0^{-1} e^u(-du)$

$-[e^u] \Big|_0^{-1}$

$-\left[\frac{1}{e} - 1\right]$

$\boxed{1 - \frac{1}{e}}$

19. $\int \frac{x}{4} e^x\, dx$

Integration by Parts

$f = \frac{x}{4}$ $\quad g' = e^x$

$f' = \frac{1}{4}$ $\quad g = e^x$

$\frac{x}{4}e^x - \int \frac{1}{4} e^x\, dx$

$\boxed{\frac{x}{4}e^x - \frac{1}{4}e^x + C}$

20. $\int \frac{1}{(2x+1)(1-x)}\, dx$

Partial Fractions!

$\left[\frac{1}{(2x+1)(1-x)} = \frac{A}{2x+1} + \frac{B}{1-x}\right](2x+1)(1-x)$

$1 = A(1-x) + B(2x+1)$

Let $x = -\frac{1}{2}$ $\quad$ Let $x = 1$

$1 = A\left(\frac{3}{2}\right)$ $\quad 1 = B(3)$

$\frac{2}{3} = A$ $\quad \frac{1}{3} = B$

$\int \frac{2/3}{2x+1} + \frac{1/3}{1-x}\, dx$

$\frac{2}{3} \cdot \frac{1}{2} \cdot \ln|2x+1| + \frac{1}{3} \cdot (-1) \cdot \ln|1-x|$

$\boxed{\frac{1}{3} \ln\left|\frac{2x+1}{1-x}\right| + C}$

21. $\int_0^1 \frac{2x}{\sqrt{x^2+1}} dx$

$u = x^2 + 1$

$\frac{du}{2x} = dx$

$\int_1^2 \frac{1}{\sqrt{u}} du$

$2\sqrt{u} \Big|_1^2$

$\boxed{2\sqrt{2} - 2}$

22. $\int_0^{\frac{\pi}{8}} \tan(2x)\sec^2(2x)\, dx$

$u = \tan(2x)$

$du = 2\sec^2(2x)dx$

$\frac{du}{2\sec^2(2x)} = dx$

$\frac{1}{2}\int_0^1 u\, du$

$\frac{1}{2}\left[\frac{u^2}{2}\right]\Big|_0^1$

$\frac{1}{4}[1-0] = \boxed{\frac{1}{4}}$

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Test Prep

23. $\int_{-1}^{1} \frac{2}{1+x^2} dx =$

$2\tan^{-1}(x) \Big|_{-1}^{1}$

$2\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right]$

$2\left[\frac{2\pi}{4}\right]$

(A) $-\pi$ (B) $-\frac{\pi}{2}$ (C) 0 (D) $\frac{\pi}{2}$ (E) π

24. $\int x\sqrt{3x}\, dx =$

$x = \sqrt{x^2}$

$\int \sqrt{3x^3}\, dx$

$\sqrt{3}\int x^{\frac{3}{2}}\, dx = \sqrt{3}\,\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$

(A) $\frac{2\sqrt{3}}{5}x^{\frac{5}{2}} + C$ (B) $\frac{5\sqrt{3}}{2}x^{\frac{5}{2}} + C$ (C) $\frac{\sqrt{3}}{2}x^{\frac{1}{2}} + C$ (D) $2\sqrt{3x} + C$ (E) $\frac{5\sqrt{3}}{2}x^{\frac{3}{2}} + C$