

Name: \_\_\_\_\_ Date: \_\_\_\_\_

**End of Unit 10 CA – Infinite Sequences and Series**

1. Let  $f$  be the function defined by  $f(x) = 3x \cos x$ . What is the coefficient of  $x^5$  in the Taylor Series for  $f$  about  $x = 0$ ?

2. Determine the number of terms required to approximate the sum of the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$  with an error less than 0.0001.

(A) 7

(B) 8

(C) 9

(D) 10

3. Find the third-degree Taylor Polynomial for the function  $f(x) = \sqrt{x}$  about  $x = 2$ .

4. What is the coefficient of  $x^3$  in the Maclaurin series for the function  $\left(\frac{1}{1-x}\right)^2$ ?

5. The function  $f$  has derivatives of all orders for all real numbers and  $f^{(4)}(x) \leq \frac{1}{2}$ . If a third-degree Taylor Polynomial for  $f$  about  $x = 0$  is used to approximate  $f$  on  $[0,1]$ . What is the Lagrange error bound for the maximum error on interval  $[0,1]$  in the approximation of  $f(1)$ ?

(A)  $\frac{1}{2}$

(B)  $\frac{1}{8}$

(C)  $\frac{1}{24}$

(D)  $\frac{1}{48}$

6. What is the alternating series error bound, if the series  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{5n+2}$  is approximated by the partial sum with 15 terms?

(A)  $\frac{1}{15}$

(B)  $\frac{1}{16}$

(C)  $\frac{1}{77}$

(D)  $\frac{1}{82}$

7.  $\max_{0 \leq x \leq 2} |f^{(5)}(x)| = 3.6$

$\max_{0 \leq x \leq 2} |f^{(6)}(x)| = 8.1$

$\max_{0 \leq x \leq 2} |f^{(7)}(x)| = 11.3$

Let  $P(x)$  be the fifth-degree Taylor Polynomial for a function  $f$  about  $x = 0$ . Information about the maximum of the absolute value of selected derivatives of  $f$  over the interval  $0 \leq x \leq 2$  is given in the table above. What is the smallest value of  $k$  for which the Lagrange error bound guarantees that  $|f(0.2) - P(0.2)| \leq k$ ?

8. Find the interval of convergence of the power series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-3)^n}{n3^n}$ .

9. What is the coefficient of  $(x-2)^4$  in the Taylor Polynomial for  $f(x) = e^{4x}$  about  $x = 2$ ?

10. A series expansion for function  $f(x) = e^{3x}$  is given by

(A)  $1 + 3x + \frac{9x^2}{2} + \frac{9x^3}{2} + \dots$

(B)  $1 + 3x + \frac{3x^2}{2!} + \frac{3x^3}{3!} + \dots$

(C)  $1 - 3x + \frac{9x^2}{2} - \frac{9x^3}{2} + \dots$

(D)  $1 - 3x + \frac{3x^2}{2!} - \frac{3x^3}{3!} + \dots$

11. Let  $f$  be the function with initial condition  $f(0) = 0$  and derivative  $f'(x) = e^{3x}$ . Write the first four nonzero terms and the general term of the Maclaurin series for  $f$ .

12. Find the radius of convergence for the power series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{6^n}$ .

### Answers to End of Unit 10 Corrective Assignment

1. $\frac{1}{8}$	2. D	3. $f(x) = \sqrt{2} + \frac{\sqrt{2}}{4}(x-2) - \frac{\sqrt{2}}{32}(x-2)^2 + \frac{\sqrt{2}}{128}(x-2)^3$	
4. 4	5. D	6. D	7. $7.2 \times 10^{-7}$
8. $0 < x \leq 6$		9. $\frac{32e^8}{3}$	10. A
11. $f(x) = x + \frac{3}{2}x^2 + \frac{3}{2}x^3 + \frac{9}{8}x^4 + \dots + \frac{3^n x^{n+1}}{(n+1)n!}$			12. 6