

End-of-Unit 10 Review – Infinite Sequences and Series**Lessons 10.10 through 10.15**

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 10.

10.10 problems

1. If the infinite series $S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n^3 - 1}$ is approximated by $S_k = \sum_{n=1}^k (-1)^{n+1} \frac{1}{2n^3 - 1}$, what is the least value of k for which the alternating series error bound guarantees that $|S - S_k| < 10^{-3}$?

- (A) 6 (B) 7 (C) 8 (D) 9

2. If the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{4n + 1}$ is approximated by the partial sum with 15 terms, what is the alternating series error bound?

- (A) $\frac{1}{15}$ (B) $\frac{1}{16}$ (C) $\frac{1}{61}$ (D) $\frac{1}{65}$

10.11 problems

3. Let $P(x) = 3 - 2x^2 + 5x^4$ be the fourth-degree Taylor Polynomial for the function f about $x = 0$. What is the value of $f^{(4)}(0)$?
4. The function f has derivatives of all orders for all real numbers with $f(2) = -2$, $f'(2) = 4$, $f''(2) = 8$, and $f'''(2) = 14$. Using the third-degree Taylor Polynomial for f about $x = 2$, what is the approximation of $f(2.2)$?

10.12 problems

5. Let f be a function that has derivatives of all orders for all real numbers and let $P_4(x)$ be the fourth-degree Taylor Polynomial for f about $x = 0$. $|f^{(n)}(x)| \leq \frac{n}{n+1}$, for $1 \leq n \leq 6$ and all values of x . Of the following, which is the smallest value of k for which the Lagrange error bound guarantees that $|f(1) - P_4(1)| \leq k$?

(A) $\frac{4}{5} \left(\frac{1}{4!} \right)$

(B) $\frac{4}{5} \left(\frac{1}{5!} \right)$

(C) $\frac{1}{6} \left(\frac{1}{4!} \right)$

(D) $\frac{1}{6} \left(\frac{1}{5!} \right)$

6. The third Maclaurin polynomial for $\sin x$ is given by $f(x) = x - \frac{x^3}{3!}$. If this polynomial is used to approximate $\sin(0.3)$, what is the Lagrange error bound?

10.13 problems

7. Find the interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1}$.

8. If the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{5^n}$ is 5, what is the interval of convergence?

10.14 problems

9. Which of the following is an expression for a function f that has the Maclaurin Series $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots + \frac{x^{2n}}{(2n)!}$?

(A) $\cos x$

(B) $e^x - \sin x$

(C) $\frac{1}{2}(e^x + e^{-x})$

(D) e^{x^2}

10. Find the Maclaurin Series for the function $f(x) = 2 \sin x^3$. Write the first four non-zero terms.

10.15 problems

11. It is known the Maclaurin series for the function $\frac{1}{1+x}$ is defined by $\sum_{n=0}^{\infty} (-1)^n x^n$. Use this fact to find the first four nonzero terms and the general term for the power series expansion for $\frac{x^2}{1+x^2}$.

12. Let $T(x) = 7 - 3(x - 3) + 5(x - 3)^2 - 2(x - 3)^3 + 6(x - 3)^4$ be the fourth-degree Taylor Polynomial for the function f about $x = 3$. Find the third-degree Taylor Polynomial for the derivative f' about $x = 3$ and use it to approximate $f'(3.3)$.