Name: ______________________________ Date:______________ Period: _____ **Review**

End-of-Unit 10 Review – Infinite Sequences and Series

Lessons 10.10 through 10.15

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 10.

10.10 problems

1. If the infinite series $S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n^3 - 1}$ is approximated by $S_k = \sum_{n=1}^{k} (-1)^{n+1} \frac{1}{2n^3 - 1}$, what is the least value of k for which the alternating series error bound guarantees that $|S - S_k| < 10^{-3}$?

$$|S - S_k| \le |a_{k+1}| < \frac{1}{1000}$$

$$\frac{1}{2(k+1)^3 - 1} < \frac{1}{1000}$$

$$1000 < 2(k+1)^3 - 1$$

$$1001 < 2(k+1)^3$$

$$500.5 < (k+1)^3$$

$$7.939 < k+1$$

$$6.939 < k$$

(A) 6 (B) 7 (C) 8 (D) 9

2. If the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{4n+1}$ is approximated by the partial sum with 15 terms, what is the alternating series error bound?

"next term" $= a_{n+1}$

16^{th} term $= \frac{1}{4(16)+1} = \frac{1}{65}$

(A) $\frac{1}{15}$ (B) $\frac{1}{16}$ (C) $\frac{1}{61}$ (D) $\frac{1}{65}$

10.11 problems

3. Let $P(x) = 3 - 2x^2 + 5x^4$ be the fourth-degree Taylor Polynomial for the function f about $x = 0$. What is the value of $f^{(4)}(0)$?

$$\frac{f^{(4)}(0)(x-0)^4}{4!} = 5x^4$$

$$f^{(4)}(0) = 5 \cdot 4! = \boxed{120}$$

4. The function f has derivatives of all orders for all real numbers with $f(2) = -2$, $f'(2) = 4$, $f''(2) = 8$, and $f'''(2) = 14$. Using the third-degree Taylor Polynomial for f about $x = 2$, what is the approximation of $f(2.2)$?

$$P_3(x) = f(2) + f'(2)x + \frac{f''(2)(x-2)^2}{2} + \frac{f'''(2)(x-2)^3}{3!}$$

$$P_3(x) = -2 + 4x + 4(x-2)^2 + \frac{7}{3}(x-2)^3$$

$$f(2.2) \approx P_3(2.2) = \boxed{-1.021}$$

10.12 problems

5. Let f be a function that has derivatives of all orders for all real numbers and let $P_4(x)$ be the fourth-degree Taylor Polynomial for f about $x = 0$. $|f^{(n)}(x)| \leq \frac{n}{n+1}$, for $1 \leq n \leq 6$ and all values of x. Of the following, which is the smallest value of k for which the Lagrange error bound guarantees that $|f(1) - P_4(1)| \leq k$?

$$R_4(x) \leq K$$

$$\frac{f^{(5)}(x)\cdot(1-0)^5}{5!} \leq K$$

$$f^5(x)\cdot\frac{1}{5!} \leq K$$

$$\frac{5}{5+1}\cdot\frac{1}{5!} \leq K$$

$$\frac{\cancel{5}}{6}\cdot\frac{1}{\cancel{5}\cdot 4!} \leq K$$

(A) $\frac{4}{5}\left(\frac{1}{4!}\right)$ (B) $\frac{4}{5}\left(\frac{1}{5!}\right)$ (C) $\frac{1}{6}\left(\frac{1}{4!}\right)$ (D) $\frac{1}{6}\left(\frac{1}{5!}\right)$

6. The third Maclaurin polynomial for $\sin x$ is given by $f(x) = x - \frac{x^3}{3!}$. If this polynomial is used to approximate $\sin(0.3)$, what is the Lagrange error bound?

$$\frac{\max[f^4(z)](0.3-0)^4}{4!} = \frac{1\cdot(0.3)^4}{4!} = \boxed{3.375 \times 10^{-4}}$$

10.13 problems

7. Find the interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1}$.

$$\lim_{n\to\infty}\left|\frac{(x-1)^{n+1+1}}{n+1+1}\cdot\frac{n+1}{(x-1)^{n+1}}\right| < 1$$

$$\lim_{n\to\infty}\left|(x-1)\cdot\frac{n+1}{n+2}\right| < 1$$

$$\lim_{n\to\infty}|x-1| < 1$$

$$-1 < x-1 < 1$$

$$0 < x < 2$$

Check endpoints!

$x=0$: $\sum_{n=0}^{\infty}\frac{1}{n+1}$ diverges

$x=2$: $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n+1}$ converges

$$\boxed{0 < x \leq 2}$$

8. If the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{5^n}$ is 5, what is the interval of convergence?

Centered at $x=0$

radius $= 5$

$$-5 < x < 5$$

Check endpoints

$x=-5$: $\sum_{n=0}^{\infty}\frac{5^n}{5^n}$ diverges

$x=5$: $\sum_{n=0}^{\infty}\frac{(-1)^n 5^n}{5^n}$ diverges

$$\boxed{-5 < x < 5}$$

10.14 problems

9. Which of the following is an expression for a function f that has the Maclaurin Series $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots + \frac{x^{2n}}{(2n)!}$?

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \frac{x^n}{n!}$ $\quad$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

(A) $\cos x$ $\quad$ (B) $e^x - \sin x$ $\quad$ (C) $\frac{1}{2}(e^x + e^{-x})$ $\quad$ (D) e^{x^2}

Series alternates signs

$\left(1 + x + \frac{x^2}{2!} + \cdots\right) - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\right)$

$1 + \frac{x^2}{2!} + \frac{2x^3}{3!} + \cdots$

no!

$\frac{1}{2}\left[\left(1 + x + \frac{x^2}{2!} + \cdots\right) + \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!}\cdots\right)\right]$

$\frac{1}{2}\left[2 + \frac{2x^2}{2!} + \frac{2x^4}{4!} + \frac{2x^6}{6!} + \cdots\right]$

Yes!

$1 + x^2 + \frac{x^4}{2!} + \cdots$

no!

10. Find the Maclaurin Series for the function $f(x) = 2\sin x^3$. Write the first four non-zero terms.

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$

$2\sin x^3 = \boxed{2x^3 - \frac{2x^9}{3!} + \frac{2x^{15}}{5!} - \frac{2x^{21}}{7!}}$ $\quad$ can be simplified

10.15 problems

11. It is known the Maclaurin series for the function $\frac{1}{1+x}$ is defined by $\sum_{n=0}^{\infty}(-1)^n x^n$. Use this fact to find the first four nonzero terms and the general term for the power series expansion for $\frac{x^2}{1+x^2}$.

$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n$

$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots + (-1)^n x^{2n}$

$x^2 \cdot \frac{1}{1+x^2} = \boxed{x^2 - x^4 + x^6 - x^8 + \cdots + (-1)^n x^{2n+2}}$

12. Let $T(x) = 7 - 3(x-3) + 5(x-3)^2 - 2(x-3)^3 + 6(x-3)^4$ be the fourth-degree Taylor Polynomial for the function f about $x = 3$. Find the third-degree Taylor Polynomial for the derivative f' about $x = 3$ and use it to approximate $f'(3.3)$.

$T_3'(x) = -3 + 10(x-3) - 6(x-3)^2 + 24(x-3)^3$

$f'(3.3) \approx T_3'(3.3) = \boxed{0.108}$