Name: Solutions Date:_______________ Period: ______ **Review**

Mid-Unit 10 Review – Infinite Sequences and Series

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 10.

1. Which of the following series converges? Use Alternating series Test

(A) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n+3}{3n}\right)$ $\lim_{n\to\infty} b_n = \frac{1}{3}$

(B) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n^2}{2\sqrt{n}}\right)$ $\lim_{n\to\infty} b_n = \infty$

(C) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{2\sqrt{n}}{n}\right)$ $\lim_{n\to\infty} b_n = 0$

b_n is decreasing

(D) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{4-n}{n}\right)$ $\lim_{n\to\infty} b_n = -1$

C

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2. What is the value of $\sum_{n=1}^{\infty}\frac{2^{n+1}}{7^n}$? $= \sum_{n=1}^{\infty}\frac{2^n\cdot 2}{7^n} = \sum_{n=1}^{\infty} 2\cdot\left(\frac{2}{7}\right)^n = \frac{a\cdot r^k}{1-r}$

$$\frac{2\cdot\left(\frac{2}{7}\right)^1}{1-\frac{2}{7}} = \frac{\frac{4}{7}}{\frac{5}{7}} = \frac{4}{7}\cdot\frac{7}{5} = \boxed{\frac{4}{5}}$$

3. Which of the following series can be used with the Limit Comparison Test to determine whether the series $\sum_{n=1}^{\infty}\frac{2^n}{3^n - n^2}$ converges or diverges?

$$\frac{2^n}{3^n} = \left(\frac{2}{3}\right)^n$$

(A) $\sum_{n=1}^{\infty}\left(\frac{3}{2}\right)^n$

(B) $\sum_{n=1}^{\infty}\frac{1}{3^n}$

(C) $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$

(D) $\sum_{n=1}^{\infty}\frac{1}{n}$

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4. **Calculator active.** Find the sequence of partial sums $S_1, S_2, S_3, S_4,$ and S_5 for the infinite series $\sum_{n=1}^{\infty}\frac{3}{2^{n-1}}$.

$S_1 = \frac{3}{2^0} = \boxed{3}$

$S_2 = \frac{3}{2} + S_1 = \boxed{4.5}$

$S_3 = \frac{3}{2^2} + S_2 = \boxed{5.25}$

$S_4 = \frac{3}{2^3} + S_3 = \boxed{5.625}$

$S_5 = \frac{3}{2^4} + S_4 = \boxed{5.8125}$

5. Verify that the infinite series $\sum_{n=1}^{\infty} \frac{3^n+1}{3^{n+2}}$ diverges by using the nth-Term Test for Divergence. Show the value of the limit.

$$\lim_{n\to\infty} \frac{3^n+1}{3^2 \cdot 3^n} = \boxed{\frac{1}{9}} \neq 0$$

Diverges

6. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{3^n}{n!}$

Ratio Test

$$\lim_{n\to\infty} \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n}$$

$$\lim_{n\to\infty} \frac{3}{n+1} = 0$$

< 1 converges

II. $\sum_{n=1}^{\infty} \frac{n}{8^n}$

Ratio Test

$$\lim_{n\to\infty} \frac{n+1}{8^{n+1}} \cdot \frac{8^n}{n}$$

$$\lim_{n\to\infty} \frac{n+1}{n(8)} = \frac{1}{8}$$

< 1 converges

III. $\sum_{n=1}^{\infty} \frac{2}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{2}{n^{3/2}}$

p-series

$p > 1$

converges

(A) I only (B) I and II only (C) I and III only (D) I, II, and III

7. For what values of x is the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n}$ conditionally convergent?

$$\lim_{n\to\infty} \left| \frac{(x+2)^{n+1}}{n+1} \cdot \frac{n}{(x+2)^n} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{n(x+2)}{n+1} \right| < 1$$

$$|x+2| < 1$$

$$-1 < x+2 < 1$$

$$-3 < x < -1$$

If $x=-3$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

Alt. harmonic converges

If $x=-1$

$\sum_{n=1}^{\infty} \frac{1}{n}$

Harmonic diverges

(A) $x > -1$ (B) $x = -3$ (C) $x = -1$ (D) $x = 3$

8. Use the Integral Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}$.

$$\int_1^{\infty} \frac{e^{1/n}}{n^2}\,dn$$

$u = \frac{1}{n}$

$du = -\frac{1}{n^2}\,dn$

$-n^2\,du = dn$

$$\int_1^0 -e^u\,du$$

$$-e^u \Big|_1^0$$

$$-e^0 - -e^1$$

$$\boxed{-1+e \quad \text{Series converges}}$$

9. Which of the following series converge?

p-series I. $\sum_{n=1}^{\infty} \frac{n^{-1}}{\sqrt{n}}$ II. $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ III. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

$u = \ln n$

$du = \frac{1}{n} dn$

$n\,du = dn$

$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

Converges

$p > 1$

Geometric

$r < 1$

Converges

Integral Test

$\int_{\ln 2}^{\infty} \frac{1}{u}\, du$

$\ln|u| \Big|_{\ln 2}^{\infty}$

$\ln(\infty) - \ln[\ln 2]$

Diverges

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

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10. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{3}{1+2^n}$ is true?

$\lim_{n \to \infty} a_n = 0$

$b_n = \frac{1}{2^n}$

$0 \le a_n \le b_n$

$\sum_{n=1}^{\infty} b_n$ converges. Geometric Series

X (A) Diverges by the nth Term test.

(B) Diverges by comparison to $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

(C) Converges by comparison to $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

(D) Diverges by comparison to $\sum_{n=1}^{\infty} \frac{1}{n}$.

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11. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n^2+3}$ is true?

Alternating

$b_n = \frac{n}{n^2+3}$

$\lim_{n \to \infty} b_n = 0$ ✓

b_n is decreasing ✓

(A) The series diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(B) The series diverges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(C) The series converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(D) The series converges by the Alternating Series Test.

12. Which of the following is required in order to apply the Integral Test to the series $\sum_{n=1}^{\infty} a_n$?

(A) $\lim_{n\to\infty} a_n = 0$ and $\sum_{n=1}^{\infty} a_n$ is a positive series.

(B) $\lim_{n\to\infty} a_n \neq 0$ and $\sum_{n=1}^{\infty} a_n$ is a convergent series.

(C) $a_n = f(n)$ and $f(x)$ is positive, continuous, and increasing on $[1, \infty)$.

(D) $a_n = f(n)$ and $f(x)$ is positive, continuous, and decreasing on $[1, \infty)$.

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13. If $a_n > 0$ for all n and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{2}{3}$, which of the following series converges? Ratio Test

(A) $\sum_{n=1}^{\infty} 3^n a_n$

$$\frac{3^{n+1} a_{n+1}}{3^n a_n}$$
$$3\left(\frac{a_{n+1}}{a_n}\right)$$
$$3\left(\frac{2}{3}\right)$$
$2 > 1$, Diverges

(B) $\sum_{n=1}^{\infty} \frac{2^n}{a_n}$

$$\frac{2^{n+1}}{a_{n+1}} \cdot \frac{a_n}{2^n}$$
$$2\left(\frac{a_n}{a_{n+1}}\right)$$
$$2\left(\frac{3}{2}\right)$$
$3 > 1$, diverges

(C) $\sum_{n=1}^{\infty} a_n \left(\frac{7}{2}\right)^n$

$$\frac{a_{n+1}\left(\frac{7}{2}\right)^{n+1}}{a_n\left(\frac{7}{2}\right)^n}$$
$$\frac{a_{n+1}}{a_n} \cdot \frac{7}{2}$$
$$\left(\frac{2}{3}\right)\left(\frac{7}{2}\right)$$
$\frac{7}{3} > 1$, diverges

(D) $\sum_{n=1}^{\infty} \frac{(a_n)^2}{3^n}$

$$\frac{(a_{n+1})^2}{3^{n+1}} \cdot \frac{3^n}{(a_n)^2}$$
$$\left(\frac{a_{n+1}}{a_n}\right)^2 \cdot \frac{1}{3}$$
$$\frac{4}{9} \cdot \frac{1}{3}$$
$\frac{4}{27} < 1$, converges

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14. The infinite series $\sum_{n=1}^{\infty} \frac{1}{7^{n+1}}$ has nth partial sum $S_n = \frac{1}{6}\left(\frac{1}{7} - \frac{1}{7^{n+1}}\right)$ for $n \geq 1$. What is the sum of the series?

$$\lim_{n\to\infty} S_n = \frac{1}{6}\left(\frac{1}{7} - 0\right) = \boxed{\frac{1}{42}}$$

15. For what value of r does the infinite series $\sum_{n=0}^{\infty} 10r^n$ equal 22? $\frac{ar^k}{1-r}$

$$\frac{10r^0}{1-r} = 22$$
$$10 = 22 - 22r$$
$$-12 = -22r$$

$$\boxed{r = \frac{6}{11}}$$

16. Determine whether the series $\sum_{n=1}^{\infty} \frac{\sin\left[\frac{(2n-1)\pi}{2}\right]}{n}$ converges absolutely, converges conditionally, or diverges.

Abs

$\sum_{n=1}^{\infty} \frac{1}{n}$

diverges (Harmonic Series)

$\sin\left(\frac{\pi}{2}\right), \sin\left(\frac{3\pi}{2}\right), \sin\left(\frac{5\pi}{2}\right), \sin\left(\frac{7\pi}{2}\right)$

$1 \quad -1 \quad 1 \quad -1$

Alt. Series Test with $b_n = \frac{1}{n}$

$\lim_{n\to\infty} b_n = 0$ ✓ $\quad b_n$ is decreasing ✓

$\boxed{\text{Converges conditionally}}$

17. Determine the convergence of the infinite p-series $\sum_{n=1}^{\infty} n^{-\pi}$.

$\frac{1}{n^{\pi}} \qquad p = \pi, \ \pi > 1$

$\boxed{\text{Convergent p-series}}$

18. The nth-Term Test can be used to determine divergence for which of the following series?

I. $\sum_{n=1}^{\infty} \frac{2}{n+1}$ II. $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{n}{4n+1}\right)$ III. $\sum_{n=1}^{\infty} \frac{n(n-2)^2}{3n^3+1}$

$\lim_{n\to\infty} a_n = 0 \qquad \lim_{n\to\infty} a_n = \pm\frac{1}{4} \qquad \lim_{n\to\infty} a_n = \frac{1}{3}$

Diverges! Diverges!

(A) III only (B) II and III only (C) I and III only (D) I, II, and III