Name: Solutions Date:__________ Period: _____ **Review**

Unit 2 Review – Differentiation: Definition & Fundamental Properties

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you should review all packets from Unit 2.

Find the average rate of change of each function on the given interval. Use appropriate units if necessary.

1. $w(x) = \ln x;\quad 1 \le x \le 7$

$$\frac{w(7)-w(1)}{7-1} = \frac{\ln 7 - \ln 1}{6} = \boxed{\frac{\ln 7}{6}}$$

2. $s(t) = -t^2 - t + 4;\ [1, 5]$
   t represents seconds
   s represents feet

$$\frac{s(5)-s(1)}{5-1} = \frac{-26-2}{4} = \boxed{-7 \text{ ft/sec}}$$

3. Find the derivative of $y = 2x^2 + 3x - 1$ by using the definition of the derivative. $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{2(x+h)^2+3(x+h)-1-[2x^2+3x-1]}{h}$$

$$\lim_{h\to 0}\frac{2(x^2+2hx+h^2)+3x+3h-1-2x^2-3x+1}{h}$$

$$\lim_{h\to 0}\frac{2x^2+4hx+2h^2+3h-2x^2}{h} = \lim_{h\to 0}\frac{h(4x+2h+3)}{h} = \boxed{4x+3}$$

4. For the function $h(t)$, h is the temperature of the oven in Fahrenheit, and t is the time measured in minutes.
   a. Explain the meaning of the equation $h(15) = 420$.

   The oven is 420 degrees Fahrenheit after 15 minutes.

   b. Explain the meaning of the equation $h'(43) = -11$.

   The oven is getting cooler by 11 degrees per minute on the 43rd minute.

Find the derivative of each function.

5. $f(x) = 4 - \frac{1}{2x^2}$

$$f(x) = 4 - \tfrac{1}{2}x^{-2}$$
$$f'(x) = x^{-3}$$
$$\boxed{f'(x) = \frac{1}{x^3}}$$

6. $g(x) = 3\sqrt{x} - \frac{6}{x^2} + 5\pi^3$

$$g(x) = 3x^{1/2} - 6x^{-2} + 5\pi^3$$
$$g'(x) = \tfrac{3}{2}x^{-1/2} + 12x^{-3}$$
$$\boxed{g'(x) = \frac{3}{2\sqrt{x}} + \frac{12}{x^3}}$$

7. $h(x) = 4e^x - 2\cos x$

$$\boxed{h'(x) = 4e^x + 2\sin x}$$

8. $s(t) = t^2 \sin(t)$

$$\boxed{s'(t) = 2t\sin(t) + t^2\cos(t)}$$

9. $d(t) = 3\sqrt{t}\ln t$

$$d'(t) = \tfrac{3}{2}t^{-1/2}\ln(t) + 3\sqrt{t}\,\frac{1}{t}$$
$$\boxed{d'(t) = \frac{3\ln(t)}{2\sqrt{t}} + \frac{3\sqrt{t}}{t}}$$

10. $y = \frac{4}{x} - \sec x$

$y = 4x^{-1} - \sec x$

$\boxed{\frac{dy}{dx} = -\frac{4}{x^2} - \sec(x)\tan(x)}$

11. $h(x) = \frac{2-x}{x+2}$

$h'(x) = \frac{(-1)(x+2) - (2-x)(1)}{(x+2)^2} = \frac{-x-2-2+x}{(x+2)^2}$

$\boxed{h'(x) = \frac{-4}{(x+2)^2}}$

Find the equation of the tangent line of the function at the given *x*-value.

12. $f(x) = -2x^3 + 3x$ at $x = -1$.

$y - y_1 = m(x - x_1)$

$f(-1) = -2(-1) + 3(-1) = -1 \leftarrow y_1$

$f'(x) = -6x^2 + 3$

$f'(-1) = -6(1) + 3 = -3 \leftarrow m$

$\boxed{y + 1 = -3(x+1)}$

13. $f(x) = 4\sin x - 2$ at $x = \pi$

$f(\pi) = 4\sin(\pi) - 2 = -2$

$f'(x) = 4\cos x$

$f'(\pi) = 4\cos(\pi) = 4(-1) = -4$

$\boxed{y + 2 = -4(x - \pi)}$

14. Find the equation for the normal line of $y = \frac{1}{2}x^2 + \frac{3}{4}x - 4$ at $x = -3$

$y(-3) = \frac{1}{2}(9) + \frac{3}{4}(-3) - 4$

$y(-3) = \frac{9}{2} - \frac{9}{4} - 4$

$y(-3) = \frac{18}{4} - \frac{9}{4} - \frac{16}{4} = -\frac{7}{4}$

y_1

$y' = x + \frac{3}{4}$

$y'(-3) = -3 + \frac{3}{4} = -\frac{12}{4} + \frac{3}{4} = -\frac{9}{4}$

m

$\boxed{y + \frac{7}{4} = \frac{4}{9}(x+3)}$

15. If $f(x) = 3\sin x - 2e^x$ find $f'(0)$. No calculator!

$f'(x) = 3\cos x - 2e^x$

$f'(0) = 3\cos 0 - 2e^0 = 3 - 2 = \boxed{1}$

A calculator is allowed on the following problems.

16. If $f(x) = x\sin(3x^2 - 2)$; find $f'(7)$.

$\boxed{f'(7) = 260.246}$

Math 8

17. If $f(x) = \csc(3x)$ at $x = 2$.

$\boxed{f'(2) = -36.899}$

18. Use the table below to estimate the value of $d'(120)$. Indicate units of measures.

t seconds	2	13	60	180	500
$d(t)$ feet	10	81	412	808	2,105

$\frac{808 - 412}{180 - 60} = \boxed{3.3 \text{ ft/sec}}$

19. Is the function differentiable at $x = 2$?

$$f(x) = \begin{cases} 3x - 3x^2 - 5, & x < 2 \\ 7 - 9x, & x \geq 2 \end{cases}$$

$3(2) - 3(2)^2 - 5 = 7 - 9(2)$

$6 - 12 - 5 = 7 - 18$

✓ $-11 = -11$

continuous

$3 - 6(2) = -9$

$3 - 12 = -9$

$-9 = -9$ ✓

$\boxed{\text{Yes!}}$

---

20. What values of a and b would make the function differentiable at $x = 4$?

$$f(x) = \begin{cases} a\sqrt{x} + bx^2 - 1, & x < 4 \\ \dfrac{16}{x} + bx, & x \geq 4 \end{cases}$$

$a(\sqrt{4}) + b(16) - 1 = \frac{16}{4} + b(4)$

$2a + 16b - 1 = 4 + 4b$

$2a = 5 - 12b$

$2(-4 - 28b) = 5 - 12b$

$-8 - 56b = 5 - 12b$

$-44b = 13$

$\boxed{b = -\frac{13}{44}}$

$\frac{a}{2\sqrt{4}} + 2b(4) = -\frac{16}{(4)^2} + b$

$\frac{a}{4} + 8b = -1 + b$

$\frac{a}{4} = -1 - 7b$

$a = -4 - 28b$

$a = -4 - 28\left(-\frac{13}{44}\right)$

$a = -4 + \frac{91}{11}$

$\boxed{a = \frac{47}{11}}$