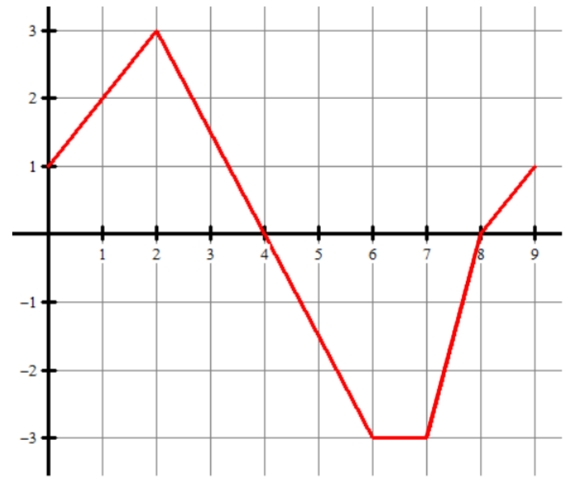
Name: Solutions Date:_______________ Period: _____

Review

Unit 4 REVIEW – Contextual Application of Differentiation

Reviews do NOT cover all material from the lessons but should remind you of key points. To be prepared, you should review all packets from Unit 4.

1. The figure shows the velocity $v = \frac{ds}{dt} = f(t)$ of a body moving along a coordinate line in meters per second.



   a) When does the body reverse direction?

   $t=4$ $t=8$

   b) When is the body moving at a constant speed?

   $(6,7)$

   c) What is the body's maximum speed?

   3 meters per second

   d) At what time interval(s) is the body slowing down?

   $(2,4)$ and $(7,8)$

Find the following. Use L'Hospital's when possible.

2. $\lim\limits_{x\to 2} \frac{x-2}{x^2-7x+10} = \frac{0}{0}$

$$\lim_{x\to 2} \frac{1}{2x-7}$$

$$\frac{1}{4-7}$$

$$\boxed{-\frac{1}{3}}$$

3. $\lim\limits_{x\to 0} \frac{3x^2}{e^x-1-x} = \frac{0}{0}$

$$\lim_{x\to 0} \frac{6x}{e^x-1} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{6}{e^x}$$

$$\boxed{6}$$

4. $\frac{d}{dx} \frac{3x-2}{5x+1}$ Quotient Rule!

$$\frac{(3)(5x+1)-(3x-2)(5)}{(5x+1)^2}$$

$$15x+3-15x+10$$

$$\boxed{\frac{13}{(5x+1)^2}}$$

5. If the length l of a rectangle is decreasing at a rate of 2 inches per minute while its width w is increasing at a rate of 2 inches per minute, which of the following must be true about the area A of the rectangle?

$\frac{dL}{dt} = -2$

$\frac{dw}{dt} = 2$

$A = LW$

$\frac{dA}{dt} = \frac{dL}{dt}w + L\frac{dw}{dt}$

$\frac{dA}{dt} = -2w + 2L$

$\frac{dA}{dt} = 2(L-w)$

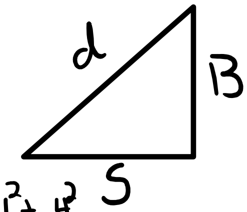
If $L > w$ then $\frac{dA}{dt}$ is positive

(A) A is always increasing. (B) A is always decreasing. (C) A is increasing only when $l > w$.

(D) A is increasing only when $l < w$. (E) A remains constant.

The following problems are calculator active.

6. Brust is riding his bicycle north away from an intersection at a rate of 15 miles per hour. Sully is driving his car towards the intersection from the west at a rate of 30 miles per hour. If Brust is 0.4 miles from the intersection, and Sully is 1 mile from the intersection, at what rate is the distance between the two of them increasing or decreasing?



$d^2 = 1^2 + .4^2$

$d = \sqrt{1.16}$

$\frac{dB}{dt} = 15$

$\frac{dS}{dt} = -30$

$B = 0.4$

$S = 1$

$\frac{dd}{dt} = ?$

$S^2 + B^2 = d^2$

$2S\frac{dS}{dt} + 2B\frac{dB}{dt} = 2d\frac{dd}{dt}$

$-60 + 12 = 2\sqrt{1.16}\,\frac{dd}{dt}$

$\boxed{\frac{dd}{dt} = -22.283 \text{ mph}}$

7. The side of a cube is increasing at a constant rate of 0.2 centimeters per second. In terms of the surface area A, what is the rate of change of the volume of the cube, in cubic centimeters per second?

$V = S^3$

$A = 6S^2$

$\frac{dS}{dt} = 0.2$

$\frac{dV}{dt} = 3S^2\frac{dS}{dt}$

$\frac{dV}{dt} = 3S^2(0.2)$

$\frac{dV}{dt} = 0.6S^2$

★ If we multiply by 10, we get A.

or $\frac{A}{10} = \frac{dV}{dt}$

(A) 0.1A (B) 0.2A (C) 0.6A

(D) 0.04A (E) 0.008A

8. The function $f(x) = (1 - \sin x)^2$ is concave up at $x = \frac{\pi}{6}$?

a. What is the estimate for $f(0.5)$ using the local linear approximation for f at $x = \frac{\pi}{6}$?

$f'(x) = 2(1 - \sin x)(-\cos x)$

$f'(\frac{\pi}{6}) = 2(1 - \frac{1}{2})(-\frac{\sqrt{3}}{2})$

$m = -\frac{\sqrt{3}}{2}$

$y_1 = f(\frac{\pi}{6}) = (1 - \frac{1}{2})^2 = \frac{1}{4}$

$y - \frac{1}{4} = -\frac{\sqrt{3}}{2}(x - \frac{\pi}{6})$

$y - \frac{1}{4} = -\frac{\sqrt{3}}{2}(0.5 - \frac{\pi}{6})$

$\boxed{f(0.5) \approx 0.270}$

b. Is it an underestimate or overestimate? Explain.

Underestimate because f(x) is concave up.