

Name: \_\_\_\_\_ Date: \_\_\_\_\_

## Unit 9 CA – Parametric Equations, Polar Coordinates, and Vector-Valued Functions

- What is the length of the curve defined by the parametric equations  $x(t) = 9 \cos t$  and  $y(t) = 9 \sin t$  for the interval  $0 \leq t \leq 2\pi$ ?
- Calculator active.** Find the area of the region inside the circle  $r = 1$  and outside the cardioid  $r = 1 - \cos \theta$ .
- If  $x(t) = 2t^3$  and  $y(t) = t^3 - t$ , what is  $\frac{d^2y}{dx^2}$  in terms of  $t$ ?
- The position of a remote-controlled vehicle moving along a flat surface at time  $t$  is given by  $(x(t), y(t))$ , with velocity vector  $v(t) = \langle 3t^2, 2t \rangle$  for  $0 \leq t \leq 3$ . Both  $x(t)$  and  $y(t)$  are measured in meters, and time  $t$  is in seconds. When  $t = 0$ , the remote-controlled vehicle is at the point  $(1, 2)$ .
  - Find the acceleration vector of the remote-controlled vehicle when  $t = 2$ .
  - Find the position of the remote-controlled vehicle when  $t = 3$ .
- Which of the following gives the length of the path described by the parametric equations  $x = 2e^{3t}$  and  $y = 3t^2 + t$  from  $0 \leq t \leq 1$ ?

A.  $\int_0^1 \sqrt{12e^{6t} + (6t + 1)^2} dt$

B.  $\int_0^1 \sqrt{4e^{6t} + (6t + 1)^2} dt$

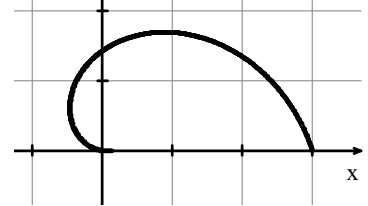
C.  $\int_0^1 \sqrt{4e^{6t} + 9t^4 + t^2} dt$

D.  $\int_0^1 \sqrt{36e^{6t} + (6t + 1)^2} dt$

6. **Calculator active.** A polar curve is given by  $r = \frac{5}{3 - \sin \theta}$ . What angle  $\theta$  corresponds on the curve with a  $y$ -coordinate of  $-1$ ?
7. If  $f$  is a vector-valued function defined by  $\langle te^t, 2t^2e^t \rangle$  then  $f''(1) = ?$
8. **Calculator active.** Find the area of the region common to the two regions bounded by the curves  $r = 6 \cos \theta$  and  $r = 2 + 2 \cos \theta$ .
9. Find the vector-valued function  $f(t)$  that satisfies the initial conditions  $f(0) = \langle 3, 0 \rangle$ , and  $f'(t) = \langle 4 \sin \frac{t}{2}, -2 \cos 2t \rangle$ .
10. If  $x = 7 \cos \theta$  and  $y = 7 \sin \theta$ , find the slope and the concavity at  $\theta = \frac{\pi}{4}$ .
11. **Calculator active.** At time  $t \geq 0$ , a particle moving in the  $xy$ -plane has velocity vector given by  $v(t) = \langle 9t^2, e^t \rangle$ . If the particle is at point  $(3, 4)$  at time  $t = 0$ , how far is the particle from the origin at time  $t = 2$ ?
12. Find the slope of the tangent line to the polar curve  $r = 2 \cos \theta - 1$  at the point where  $\theta = \frac{3\pi}{2}$ .

13. Find the slope of the tangent line to the curve defined parametrically by  $x(t) = 2 \cos t$  and  $y(t) = 3 \sin^2 t$  at  $t = \frac{\pi}{3}$ .

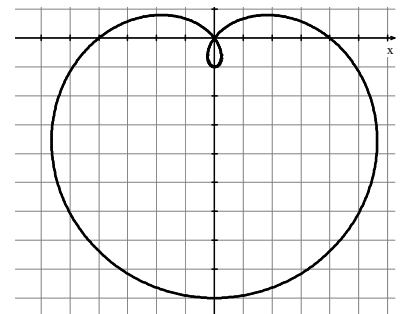
14. **Calculator active.** The graph shows the polar curve  $r = 3 - \theta$  for  $0 \leq \theta \leq \pi$ . What is the area of the region bounded by the curve and the  $x$ -axis?



15. At time  $t$ ,  $0 \leq t \leq 2\pi$ , the position of a particle moving along a path in the  $xy$ -plane is given by the vector-valued function,  $f(t) = \langle \cos 2t, \sin 4t \rangle$ . Find the slope of the path of the particle at time  $t = \frac{\pi}{4}$ .

16. Find an equation for the line tangent to the curve given by the parametric equations  $x(t) = t^2 + 1$  and  $y(t) = t^3 + t + 1$ , when  $t = 2$ .

17. **Calculator active.** Find the total area enclosed by the inner loop of the polar curve  $r = 4 - 5 \sin \theta$ , shown in the figure.



### Answers to Unit 9 Corrective Assignment

1. $18\pi$	2. 1.215 or $2 - \frac{\pi}{4}$	3. $\frac{1}{18}t^{-5}$	4a. $\langle 12, 2 \rangle$ 4b. $\langle 28, 11 \rangle$	5. D
6. $\theta = 5.435$ or $\theta = 3.990$	7. $\langle 3e, 14e \rangle$	8. 15.708 or $5\pi$	9. $\langle -8 \cos \frac{t}{2} + 11, -\sin 2t \rangle$	
10. Slope: $-1$ , Concave Down	11. 28.930	12. 2	13. $\frac{dy}{dx} = -\frac{3}{2}$	
14. 4.500	15. 2	16. $y = \frac{13}{4}x - \frac{21}{4}$	17. 0.340	