Corrective Assignment

Name: ______________________________ Date:______________

End-of-Unit 8 CA – Applications of Integration

1. Find the positive number(s) b such that the average value of $y = 2 + 7x - x^3$ on the interval $[0, b]$ is equal to 2.

2. A particle moves along the x-axis with a velocity of $v(t) = 1 - \sin t$. At $t = \pi$ seconds the position of the particle is π inches. What is the position of the particle at $t = \frac{3\pi}{2}$?

3. **Calculator active.** Revolve the region bounded by the graphs of $y = x^2$, $x = 3$, and $y = 0$ about the line $x = 3$. Find the volume of the solid.

4. **Calculator active.** Revolve the region bounded by the graphs of $y = 2 - x^2$ and $y = -2$ about the line $y = -2$. Find the volume of the solid.

5. **Calculator active.** What is the area of the region in the first quadrant enclosed by the graphs of $y = 2 - x^2$, $y = 3\sin x$, and the y-axis?

(A) 0.591 (B) 0.604 (C) 0.982 (D) 1.281 (E) 1.924

6. Find an integral that represents the length of the curve $y = \frac{1}{x^2}$ from $x = 1$ to $x = 3$. Do Not Evaluate.

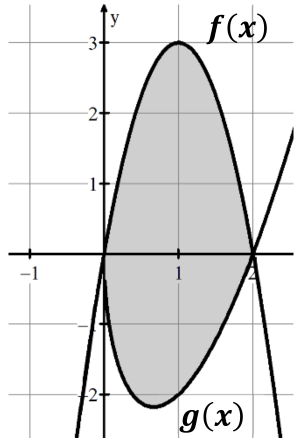
7. **Calculator active.** A storm has washed away sand from a beach, causing the edge of the water to get closer to a nearby road. The rate at which the distance between the road and the edge of the water was changing during the storm is modeled by $r(t) = e^{-\sin t}$ feet per hour, t hours after the storm began. The edge of the water was 100 feet from the road when the storm began. If the storm lasted 3 hours, how far is the water from the road after the storm?

8. If the region enclosed by the y-axis, the line $y = 2$, and the curve $y = \sqrt[3]{x}$ is revolved about the y-axis, the volume of the solid generated is

(A) π (B) 4π (C) 8π (D) $\frac{64\pi}{7}$ (E) $\frac{128\pi}{7}$

9. Find the arc length for $y = \frac{1}{3}(x^2 + 2)^{\frac{3}{2}}$ for $x = 0$ to $x = 4$.

10. **Calculator active.**



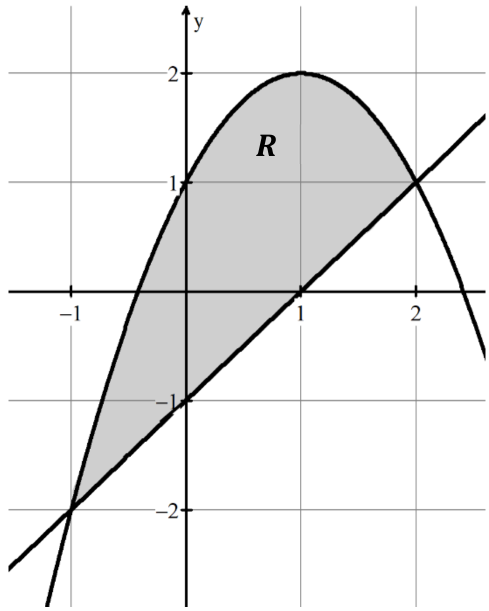
Let f and g be the functions given by $f(x) = 3x(2 - x)$ and $g(x) = 2(x - 2)\sqrt{x}$. The graphs of f and g are shown in the figure above.

a. Find the area of the shaded region enclosed by the graphs of f and g.

b. Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 3$.

c. Let h be the function given by $h(x) = kx(2 - x)$ for $0 \le x \le 2$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value k for which the volume of this solid is equal to 20. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

11. **Calculator active.**



Let R be the region bounded by the graphs of $y = x - 1$ and $y = -(x-1)^2 + 2$, as shown in the figure above.

a. Find the area of R.

b. The horizontal line $y = \frac{3}{2}$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part R that is above this horizontal line. Do this with respect to x, not respect to y, even though respect to y is much easier.

c. The region R is the base of a solid. For this solid, each cross section perpendicular to the y-axis is a square. Find the volume of this solid.

d. The region R models the surface of a small pond. At all points in R at a distance y from the x-axis, the depth of water is given by $d(x) = 2 - x$. Find the volume of water in the pond.

Answers to End-of-Unit 8 Corrective Assignment

1. $\sqrt{14}$	2. $\frac{3\pi}{2}+1$	3. $\int_0^9 \pi\left(3-\sqrt{y}\right)^2\,dy \approx 42.4115$	4. $\int_{-2}^{2} \pi(-x^2+4)^2\,dx \approx 107.233$

5. B	6. $\int_1^3 \sqrt{1+\frac{4}{x^6}}\,dx$	7. $100-\int_0^3 r(t)\,dt = 98.3858$ feet	8. E	9. $\frac{76}{3}$

10a. $A=\int_0^2 \left(f(x)-g(x)\right)dx = 7.0169$

10b. $V=\pi\int_0^2 \left[\left(3-g(x)\right)^2-\left(3-f(x)\right)^2\right]dx = 118.8629$	10c. $V=\int_0^2 (kx(2-x)-\left(2(x-2)\sqrt{x}\right)^2\,dx = 20$
11a. $A=\int_{-1}^{2}\left(-(x-1)^2+2-(x-1)\right)dx = 4.5$	11b. $\frac{3}{2}=-(x-1)^2+2$ at $a=$ 0.2928932 and $b=$ 1.7071068. $A=\int_a^b\left[-(x-1)^2+2-\frac{3}{2}\right]dx$

11c. $V=\int_{-2}^{1}\left[(y+1)-\left(1-\sqrt{2-y}\right)\right]^2 dy+\int_1^2\left[\left(1+\sqrt{2-y}\right)-\left(1-\sqrt{2-y}\right)\right]^2 dy = 6.3667$

11d. $V=\int_{-1}^{2}(2-x)[-(x-1)^2+2-(x-1)]\,dx = 6.75$