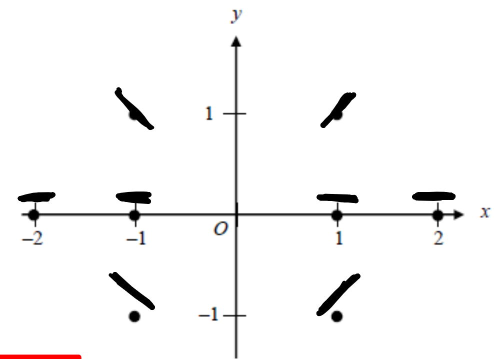
Name: Solutions Date:______________ Period: _____ **Review**

Unit 7 Review – Differential Equations

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 7.

1. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x}$, where $x \neq 0$.

   a. On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.



   b. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = -1$.

$y^{-2}dy = \frac{1}{x}dx$

$-\frac{1}{y} = \ln|x| + C$

$1 = C$

$-\frac{1}{y} = \ln|x| + 1$

$\boxed{y = -\frac{1}{\ln|x| + 1}}$

   c. Write an equation for the tangent line to the curve $y = f(x)$ through the point $(1, -1)$. Then use your tangent line equation to estimate the value of $f(1.2)$.

$\frac{dy}{dx}\Big|_{(1,-1)} = \frac{1}{1} = 1$

$y + 1 = 1\cdot(x - 1)$

$y + 1 = (1.2 - 1)$

$y + 1 = 0.2$

$\boxed{y = -0.8}$

2. The rate of change of the volume, $V(t)$, of water in a swimming pool is directly proportional to the cube root of the volume. If $V = 27\text{ ft}^3$ when $\frac{dV}{dt} = 5$, what is a differential equation that models this situation?

$\frac{dV}{dt} = k\sqrt[3]{V}$

$5 = k\sqrt[3]{27}$

$5/3 = k$

$\boxed{\frac{dV}{dt} = \frac{5}{3}\sqrt[3]{V}}$

3. The table below gives the values of f', the derivative of f. If $f(4.2) = 5$, what is the approximation to $f(4.6)$ obtained by using Euler's method with 2 steps of equal size?

x	4	4.2	4.4	4.6	4.8
$f'(x)$	0.2	0.3	0.5	0.61	0.73

x	y	y'	new y
4.2	5	0.3	$5 + 0.3(0.2) = 5.06$
4.4	5.06	0.5	$5.06 + 0.5(0.2) = 5.16$
4.6	5.16		

$y = y_1 + y'\cdot\Delta x$

$\Delta x = \frac{4.6 - 4.2}{2} = 0.2$

$\boxed{f(4.6) \approx 5.16}$

Find the general solution of the differential equation.

4. $\frac{dy}{dx} = \frac{2x}{y}$

$y\,dy = 2x\,dx$

$\frac{y^2}{2} = x^2 + C_1$

$y^2 = 2x^2 + C_2$

$\boxed{y = \pm\sqrt{2x^2 + C}}$

5. $\frac{dy}{dx} = x(y+4)$

$\frac{1}{y+4}dy = x\,dx$

$\ln|y+4| = \frac{x^2}{2} + C_1$

$y+4 = e^{\frac{1}{2}x^2 + C_1}$

$\boxed{y = Ce^{\frac{x^2}{2}} - 4}$

For each differential equation, find the particular solution that passes through the given point.

6. $\frac{dy}{dx} = \frac{18}{6x+3} + \frac{4}{x^3}$; $\left(-\frac{1}{3}, -15\right)$

$u = 6x+3$

$\frac{du}{6} = dx$

$y = 18 \cdot \frac{1}{6}\ln|6x+3| - \frac{4}{2x^2} + C$

$-15 = 3\ln|-2+3| - \frac{4}{2(\frac{1}{9})} + C$

$-15 = 0 - 4(\frac{9}{2}) + C$

$-15 = -18 + C$

$3 = C$

$\boxed{y = 3\ln|6x+3| - \frac{2}{x^2} + 3}$

7. $\frac{dy}{dx} = 2y$ and $y = -0.2$ when $x = 0$

$\frac{1}{y} = 2\,dx$

$\ln|y| = 2x + C$

$|y| = Ce^{2x}$

$0.2 = C$

$|y| = 0.2e^{2x}$

$\boxed{y = -0.2e^{2x}}$

8. A population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 12 years, then what is the value of k?

$2C = Ce^{k(12)}$

$\ln 2 = 12k$

$\boxed{k = \frac{\ln 2}{12}}$

9. The number of people in a store is modeled by a function F that satisfies the logistic differential equation $\frac{dF}{dt} = \frac{1}{500}F(100-F)$, where t is in hours and $F(0) = 10$. What is the greatest rate of change, in people per hour, of the number of people in the store?

$F' = \frac{1}{5}F - \frac{1}{500}F^2$

$F'' = \frac{1}{5} - \frac{1}{250}F$

$0 = \frac{1}{5} - \frac{1}{250}F$

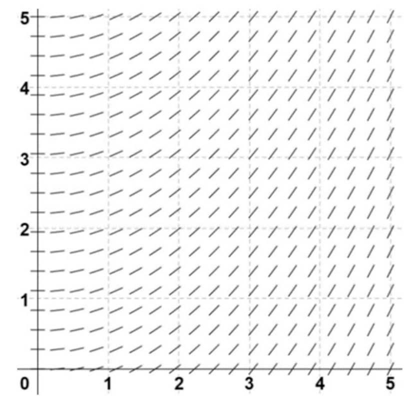
$50 = F$

$F'(50) = \frac{1}{500}(50)(100-50)$

$\frac{1}{10}(50)$

$\boxed{5 \text{ people per hour}}$

10. Explain why the following slope field cannot represent the differential equation $\frac{dy}{dt} = 0.4y$



One possible answer: When $y = 0$, $\frac{dy}{dt} = 0$. However, in the slope field, the slopes of the line segments for $y = 0$ are nonzero.

11. Given that $y = f(t)$ is a solution to the logistic differential equation $\frac{dy}{dt} = \frac{y}{5} - \frac{y^2}{1500}$, where t is time in years. What is $\lim_{t\to\infty} f(t)$?

$y' = \frac{1}{5}y\left(1 - \frac{y}{300}\right)$

limiting value is 300.

$\boxed{300}$

12. For what value of k, if any, will $y = k\cos(2x) + 3\sin(4x)$ be a solution to the differential equation $y'' + 16y = -6\cos(2x)$?

$y' = -2k\sin(2x) + 12\cos(4x)$

$\underbrace{y'' = -4k\cos(2x) - 48\sin(4x)}_{y''} + \underbrace{16[k\cos(2x) + 3\sin(4x)]}_{16y} = -6\cos(2x)$

$12k\cos(2x) = -6\cos(2x)$

$\boxed{k = -\frac{1}{2}}$

13. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 3x + y$ with initial condition $f(0) = 1$. What is the approximation for $f(0.5)$ obtained using Euler's method with 2 steps of equal length, starting at $x = 0$?

$y = y_1 + y' \cdot \Delta x$

$\Delta x = \frac{0.5-0}{2} = 0.25$

x	y	y'	new y
0	1	$3(0)+1 = 1$	$1 + 1 \cdot 0.25 = 1.25$
0.25	1.25	$3(.25)+1.25 = 2$	$1.25 + 2(0.25) = 1.75$
0.5	1.75		

$\boxed{f(0.5) \approx 1.75}$