Area and Volume

AP Calculus Free Response Problems
2002 – current year

2002 Form A #1 [calculator allowed]

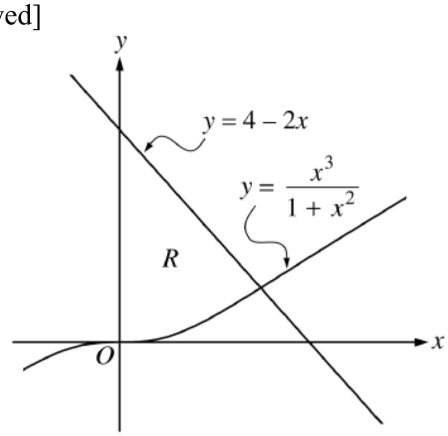
1. Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

   (a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

   (b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

   (c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

2002 Form B #1 [calculator allowed]



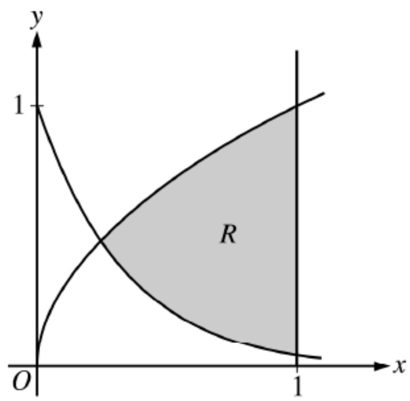
1. Let R be the region bounded by the y-axis and the graphs of $y = \frac{x^3}{1 + x^2}$ and $y = 4 - 2x$, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

2003 Form A #1 [calculator allowed]



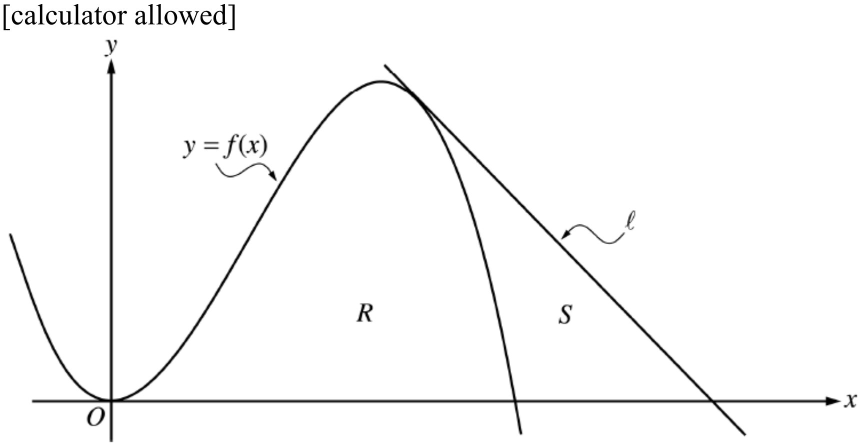
1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

2003 Form B #1 [calculator allowed]



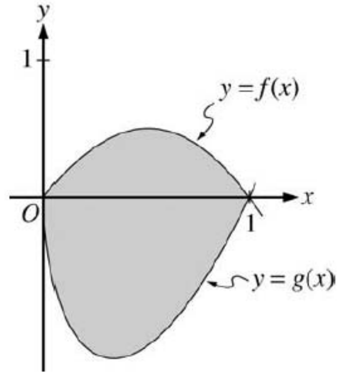
1. Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.

   (a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when R is revolved about the x-axis.

2004 Form A #2 [calculator allowed]



2. Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

   (a) Find the area of the shaded region enclosed by the graphs of f and g.

   (b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

   (c) Let h be the function given by $h(x) = kx(1-x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

2004 Form B #1 [calculator allowed]

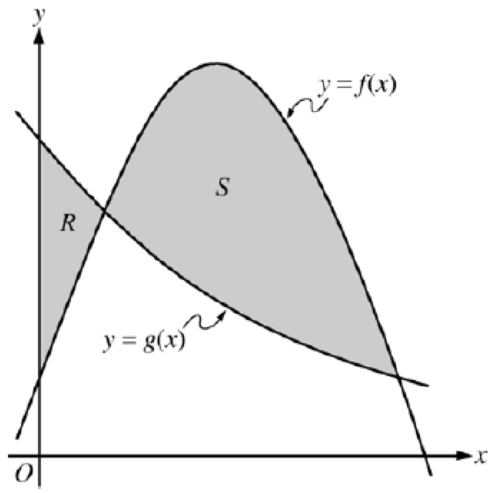
1. Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line $x = 10$, and the x-axis.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 3$.

   (c) Find the volume of the solid generated when R is revolved about the vertical line $x = 10$.

2005 Form A #1 [calculator allowed]



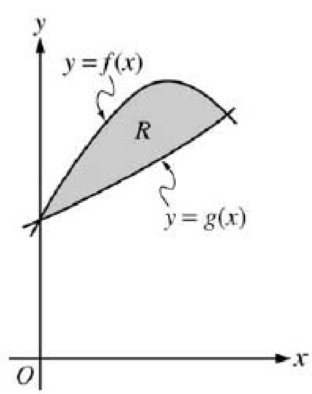
1. Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

2005 Form B #1 [calculator allowed]



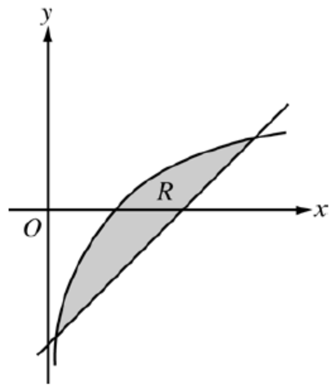
1. Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

2006 Form A #1 [calculator allowed]



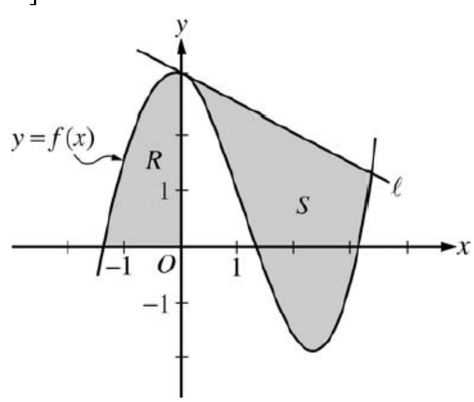
1. Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

   (c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.

2006 Form B #1 [calculator allowed]



1. Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and line ℓ, the line tangent to the graph of f at $x = 0$, as shown above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

   (c) Write, but do not evaluate, an integral expression that can be used to find the area of S.

2007 Form A #1 [calculator allowed]

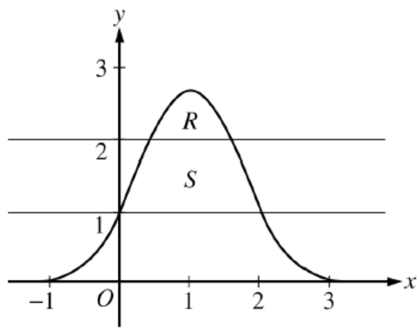
1. Let R be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1 + x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

2007 Form B #1 [calculator allowed]



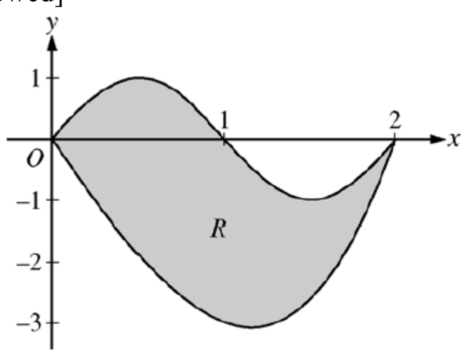
1. Let R be the region bounded by the graph of $y = e^{2x - x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x - x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

2008 Form A #1 [calculator allowed]



1. Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

(a) Find the area of R.

(b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

(d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

2008 Form B #1 [calculator allowed]

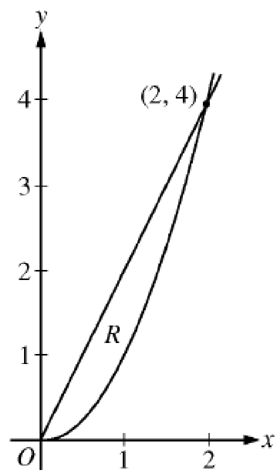
1. Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

2009 Form A #4 [no calculator!]



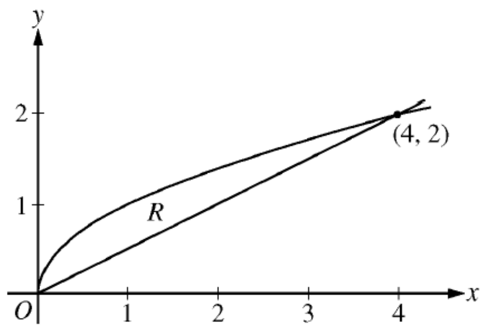
4. Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

   (a) Find the area of R.

   (b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

   (c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

2009 Form B #4 [no calculator!]



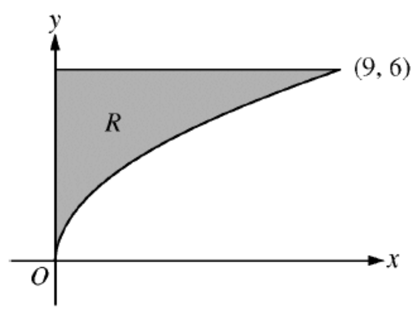
4. Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.

(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are squares. Find the volume of this solid.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

2010 Form A #4 [no calculator]



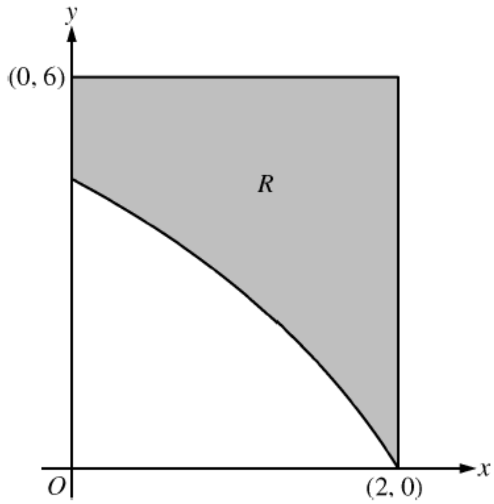
4. Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

2010 Form B #4 [no calculator]



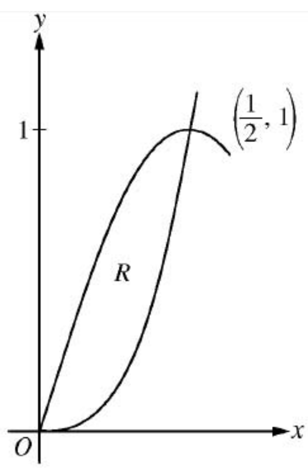
1. In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

2011 Form A #3 [calculator allowed]



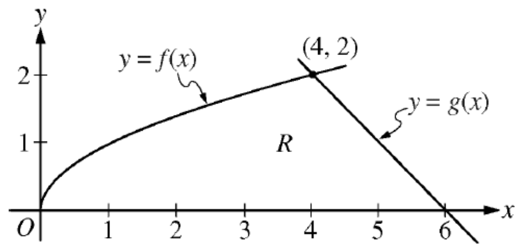
3. Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

   (a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

   (b) Find the area of R.

   (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

2011 Form B #3 [calculator allowed]



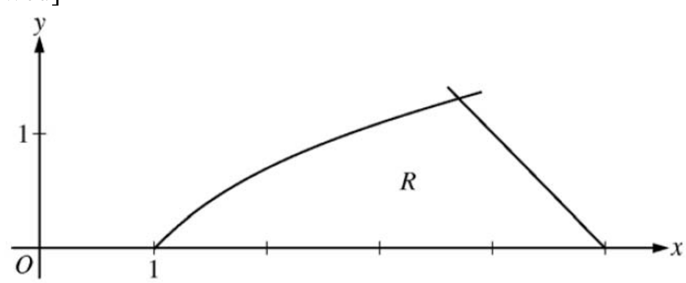
3. The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure above.

   (a) Find the area of R.

   (b) The region R is the base of a solid. For each y, where $0 \le y \le 2$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

   (c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

2012 #2 [calculator allowed]



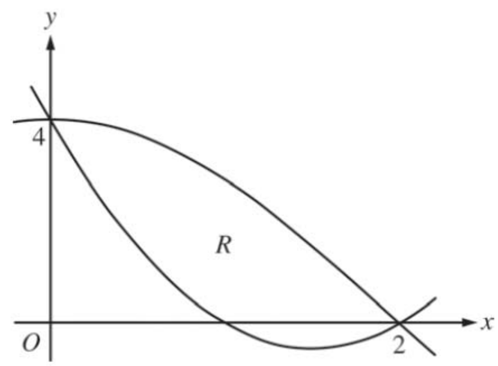
2. Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

   (a) Find the area of R.

   (b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

   (c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

2013 #5 [no calculator]



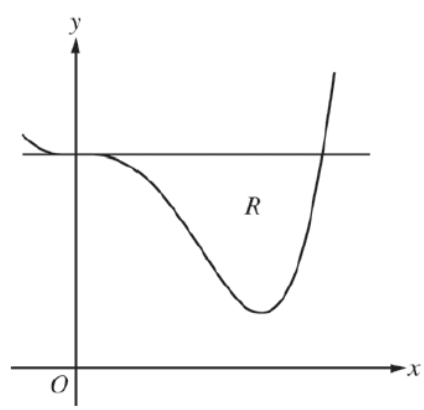
5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

2014 #2 [calculator allowed]



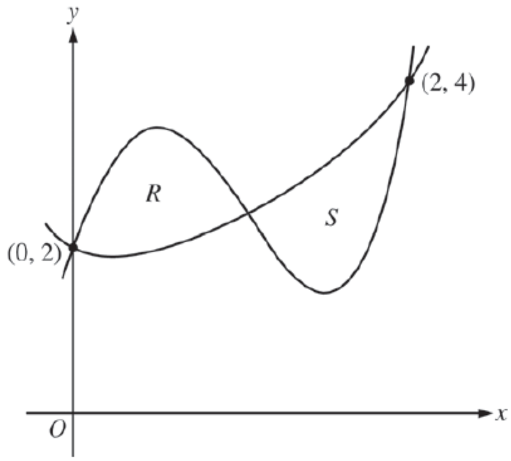
2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

2015 #2 [calculator allowed]



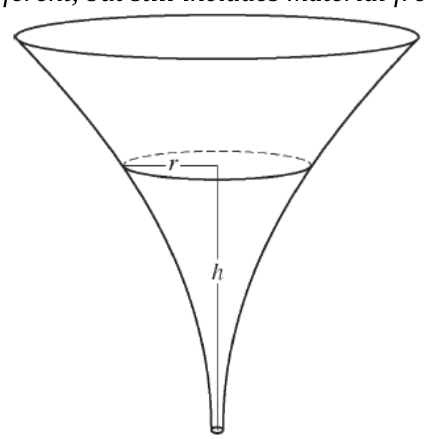
2. Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

   (a) Find the sum of the areas of regions R and S.

   (b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

   (c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.

2016 #5 [*this one is a little different, but still includes material from our Unit 11*] No calculator



5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

   (a) Find the average value of the radius of the funnel.

   (b) Find the volume of the funnel.

   (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

2017 – NONE WERE ON THE EXAM THIS YEAR!!