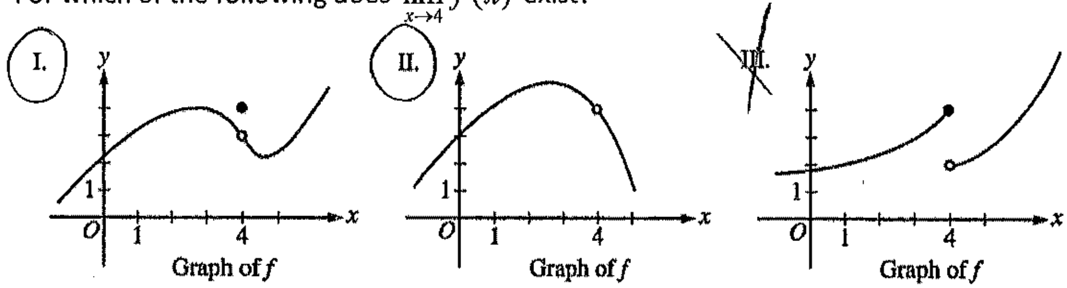
AP Calculus AB
Fall Final Review

Name Key!
Date________ Hour________

CALCULATOR REVIEW

1. For which of the following does $\lim_{x\to 4} f(x)$ exist?



2. What is the average rate of change of y = $\frac{\cos x}{x^2+x+2}$ on the closed interval [-2, 2]?

$(-2, -0.104)$ $(2, -0.052)$

$\frac{-0.052 - -0.104}{2 - -2} = \frac{0.052}{4}$

$\boxed{= 0.013}$

3. $\lim_{x\to 0} \frac{e^x - \cos x - 2x}{x^2 - 2x}$ $= 0.5$

4. If $\lim_{x\to c} f(x) = -\frac{1}{2}$ and $\lim_{x\to c} g(x) = \frac{2}{3}$, find $\lim_{x\to c} \frac{f(x)}{g(x)}$. $\frac{-\frac{1}{2}}{\frac{2}{3}} = \boxed{-\frac{3}{4}}$

5. An object is dropped from the top of a tower. Its height, in meters, above the ground after t seconds is given by the equation $y = 300 - 4.9t^2$. Give answers with correct units.

(a) What is the height of the object after 3 seconds? $\boxed{255.9 \text{ meters}}$

(b) What is the average speed of the object over the first 3 seconds? $\frac{300 - 255.9}{0 - 3} = \frac{44.1}{-3} = \boxed{-14.7 \text{ m/sec}}$

(c) What is the instantaneous speed of the object at 3 seconds? $\boxed{-29.4 \text{ m/sec}}$

(d) Write the equation of the tangent line to the graph of y when t = 3.

$y - 255.9 = -29.4(t - 3)$

6. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 2 + 4.1\cos(0.8t)$. What is the acceleration of the particle at time t = 3? $v'(3) = -2.215$

7. If $f(x) = \ln(x + 4 + e^x)$, then $f'(0)$ is?

$f'(0) = 0.4$

8. If f is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

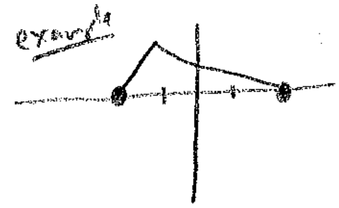
II. $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

III. $\lim_{x\to a} \frac{f(x+h)-f(x)}{h}$

9. The function f is continuous on [-2, 2] and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following must be true?

no horizontal tangents

example



(A) For $-2 < k < 2$, $f'(k) > 0$.

(B) For $-2 < k < 2$, $f'(k) < 0$.

(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

10. A rock is thrown straight into the air. Its height, in meters, above the ground after t seconds is given by the equation $s(t) = 32t - 4.9t^2$. Show your work and give answers with correct units.

(a) What is the height of the rock after 3 seconds? $s(3) = 51.9$ meters

(b) What is the average velocity of the rock over the first 3 seconds? $\frac{51.9-0}{3-0} = 17.3$ m/sec

(c) What is the instantaneous velocity of the rock at 3 seconds? $s'(3) = 2.6$ m/sec

(d) What is the maximum height of the object and how long does it take to fall back to the ground?

$s'(t) = 0$ or graph to find max

$\boxed{52.244 \text{ meters}}$

3.265 seconds it is at max height

~~6.53 seconds it is on ground~~

3.265 seconds

11. Let f be the function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at (x, f(x)) equal to 4?

$f'(x) = 2e^{4x^2}(8x)$

use calc to find intersection → $4 = 2e^{4x^2}(8x)$

$\boxed{x = 0.209}$

12. The graph of f', the derivative of the function f, is shown to the right. Which of the following statements is true?

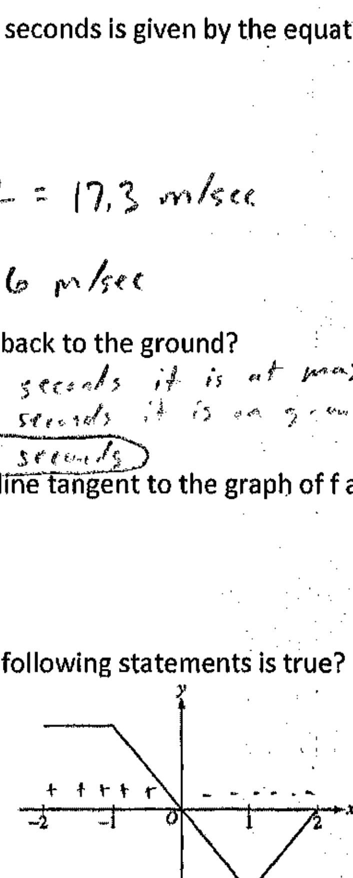
(A) f is decreasing for $-1 \le x \le 1$.

(B) f is increasing for $1 \le x \le 2$.

(C) f is not differentiable at $x = -1$ and $x = 1$.

(D) f is increasing for $-2 \le x \le 0$.

(E) f has a local minimum at $x = 0$.



Graph of f'

13. Let f be a differentiable function such that f(3) = 2 and f'(3) = 5. If the tangent line to the graph of f at x = 3 is used to find an approximation to a zero of f, find that approximation.

$y-2=5(x-3)$ $\quad$ $0=5x-13$

$y=5x-13$ $\quad$ $13=5x$

$\frac{13}{5}=x$

14. Let f be the function given by $f(x) = x^3 - 5x^2 + 3x + k$, where k is a constant.

Graph with $k=0$

(a) On what intervals is f(x) increasing? $f'(x)=3x^2-10x+3$

$(-\infty, \frac{1}{3})\ (3,\infty)$

$3x^2-10x+3=0$

$(3x-1)(x-3)=0$

$x=\frac{1}{3},\ 3$

max $\quad$ min

(b) On what intervals is the graph of f(x) concave downward?

$(-\infty, \frac{5}{3})$ $\quad$ $f''(x)=6x-10$

$6x-10=0$

$x=\frac{10}{6}=\frac{5}{3}$

POI

(c) Find the value of k for which f(x) has 11 as its relative minimum.

$11=x^3-5x^2+3x+k$

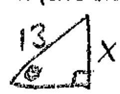
$11=3^3-5(3)^2+3(3)+k$

$\boxed{k=20}$

15. The radius of a circle is increasing at a constant rate of 0.4 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 25π meters? $A=\pi r^2$ $\quad$ $\frac{dA}{dt}=2\pi\left(\frac{25}{2}\right)(0.4)$

$\frac{dr}{dt}=0.4\ m/sec$ $\quad$ $\frac{dA}{dt}=?$ $\quad$ $C=25\pi\ m$ $\quad$ $C=2\pi r$, $25\pi=2\pi r$, $\frac{25}{2}=r$ $\quad$ $\frac{dA}{dt}=2\pi r\frac{dr}{dt}$ $\quad$ $\boxed{\frac{dA}{dt}=10\pi\ m^2/sec}$

16. In the right triangle with a hypotenuse of 13, if θ increases at a constant rate of 2 radians per minute, at what rate is x (the side opposite θ) increasing in units per minute when x equals 5 units?



$\frac{d\theta}{dt}=2\ rad/min$ $\quad$ $x=5$ $\quad$ $\frac{dx}{dt}=?$

$\sin\theta=\frac{x}{13}$ $\quad$ $\sin\theta=\frac{5}{13}$ $\quad$ $\theta=0.394$

$13\sin\theta=x$

$13\cos\theta\frac{d\theta}{dt}=\frac{dx}{dt}$

$13\cos(0.394)\,2=\frac{dx}{dt}$

$\boxed{\frac{dx}{dt}=24.007\ units/min}$

17. Let f be the function with derivative given by $f'(x) = \cos(x^2 + 1)$. How many relative extrema does f have on the interval $1 < x < 5$?

$\cos(x^2+1)=0$ $\quad$ $\boxed{7}$

graph it!

18. The position of a particle moving along the x-axis is given by the function $x(t) = e^t + t\,e^t$. What is the average velocity of the particle from time t = 0 to time t = 5?

$(0,1)\ (5, 890.478)$ $\quad$ $\frac{890.478-1}{5-0}=\boxed{177.895}$

19. Consider the curve defined by $-8x^2 + 5xy + y^3 = -125$

(a) Find dy/dx.

$-16x+5[(1)(y)+(x)(y')]+3y^2y'=0$

$-16x+5y+5xy'+3y^2y'=0$

$5xy'+3y^2y'=16x-5y$

$y'(5x+3y^2)=16x-5y$

$\boxed{\frac{dy}{dx}=\frac{16x-5y}{5x+3y^2}}$

(b) Write an equation for the line tangent to the curve at (3, -1).

$\boxed{y+1=\frac{53}{18}(x-3)}$

(c) There is a number k such that (3.2, k) is on the curve. Using the tangent line in part (b), approximate the value of k.

$k+1=\frac{53}{18}(3.2-3)$ $\quad$ $\boxed{k=-0.4\overline{1}}$

(d) Write an equation that can be solved to find the actual value of k such that (3.2, k) is on the curve.

(e) Solve the equation in part (d) for the value of k.

$-8x^2+5xy+y^3=-125$

$-8(3.2)^2+5(3.2)k+k^3=-125$

$k^3+16k+43.08=0$

$\boxed{k=-2.107}$

20. If $y = 5^x + 4x - 2$, Find dy/dx.

$\frac{dy}{dx}=5^x\ln 5+4$

NON-CALCULATOR REVIEW

1. $\lim_{x\to\infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)}$ $= \frac{6x-2x^2-3+x}{x^2+3x-1x-3} = \frac{-2x^2+7x-3}{x^2+2x-3} = \boxed{-2}$

2. What are all the horizontal asymptotes of the graph of $y = \frac{5+2^x}{1-2^x}$ in the xy-plane?

$\lim_{x\to\infty} \frac{5+2^x}{1-2^x} = \frac{5+2^\infty}{1-2^\infty} = \frac{\infty}{\infty}$

l'hopital's rule

$\lim_{x\to\infty} \frac{2^x \ln 2}{-2^x \ln 2} = \frac{1}{-1} = -1$

$\lim_{x\to-\infty} \frac{5+2^x}{1-2^x} = \frac{5+2^{-\infty}}{1-2^{-\infty}} = \frac{5+\frac{1}{2^\infty}}{1-\frac{1}{2^\infty}} = \frac{5}{1}$

0 0

$\boxed{\text{HA: } y=1 \text{ and } y=5}$

3. $\lim_{x\to\infty} \frac{1x^3-2x^2+3x-4}{4x^3-3x^2+2x-1}$ $\frac{1}{4}$

4. $\lim_{x\to 0} \frac{5x^4+8x^2}{3x^4-16x^2}$ $= \frac{\cancel{5(0)^4+8(0)}}{\cancel{3(0)^4-16(0)}}$ 0 factor out x squared should get -0.5

5. $\lim_{\theta\to 0} \frac{1-\cos\theta}{2\sin^2\theta}$ $= \frac{1-\cos(0)}{2\sin^2(0)} = \frac{1-1}{2(0)} =$ Bummer so, $\frac{1-\cos\theta}{2(1-\cos^2\theta)} = \frac{\cancel{1-\cos\theta}}{2(1+\cos\theta)(\cancel{1-\cos\theta})} = \frac{1}{2(1+\cos(0))} = \frac{1}{2(1+1)} = \frac{1}{4}$

or l'hopital's rule! $\frac{\sin\theta}{4\sin\theta(\cos\theta)} = \frac{1}{4(\cos 0)} = \frac{1}{4}$

6. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2-4}{x+2}$ when $x \neq -2$, then f(-2) = $\boxed{-4}$

$\frac{\cancel{(x+2)}(x-2)}{\cancel{x+2}} = (-2-2) = -4$

7. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4 \end{cases}$ then $\lim_{x\to 2} f(x)$ is DNE

$\lim_{x\to 2^-} \ln x = \ln 2$ $\lim_{x\to 2^+} x^2 \ln 2 = 4\ln 2$

$\ln 2 \neq 4\ln 2$

$\lim_{x\to 2^-} \neq \lim_{x\to 2^+}$

8. Let f be the function defined by $f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5-x & \text{for } 3 < x \leq 5 \end{cases}$

(a) Is f continuous at x = 3? Explain why or why not.

$\sqrt{x+1} = 5-x$
$\sqrt{3+1} = 5-3$
$2 = 2$ ✓ Yes, it is continuous!

(b) Find the average rate of change of $f(x)$ on the closed interval [0, 3]. $(0,1)\ (3,2)$ so $\frac{2-1}{3-0} = \frac{1}{3}$

(c) Suppose the function g is defined by $g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ mx+2 & \text{for } 3 < x \leq 5 \end{cases}$ where k and m are constants. If g is continuous at x = 3, what is the value of k when m = 2?

$K\sqrt{x+1} = mx+2$ when $x=3$ and $m=2$

$K\sqrt{3+1} = 2(3)+2$

$2K = 8$

$\boxed{K=4}$

9. If $y = \frac{3}{4+x^2}$, then $\frac{dy}{dx} =$ rewrite → $3(4+x^2)^{-1}$

$-3(4+x^2)^{-2}(2x) = \boxed{\frac{-6x}{(4+x^2)^2}}$

10. $\lim_{x\to\infty} \frac{4(x^3-5x^2+x-4)}{4x^3-3x^2+5x-3}$ $= \boxed{1}$

11. If the line tangent to the graph of the function f at the point (1, 7) passes through the point (-2, -2), then $f'(1)$ is

$m = \frac{7--2}{1--2} = \frac{9}{3} = 3$ $\boxed{f'(1) = 3}$

12. $\frac{d}{dx}\left(\frac{1}{x^3} - \frac{1}{x} + x^2\right)$ at x = -1 is $\xrightarrow{\text{rewrite}} x^{-3} - x^{-1} + x^2$

$f'(x) = -3x^{-4} + x^{-2} + 2x$

$f'(-1) = \frac{-3}{(-1)^4} + \frac{1}{(-1)^2} + 2(-1)$ $\boxed{f'(-1) = -4}$

$-3 + 1 - 2$

13. If $(x) = \sqrt{2x}$, then $f'(2) =$

$f(x) = (2x)^{1/2}$

$f'(x) = 1/2(2x)^{-1/2}(2)$ $\rightarrow$ $f'(x) = \frac{1}{\sqrt{2x}}$

$f'(2) = \frac{1}{\sqrt{2(2)}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$ $\boxed{f'(2) = 1/2}$

14. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

$V(t) = 0$

$x'(t) = 6t^2 - 42t + 72$

$0 = 6(t^2 - 7t + 12)$

$0 = 6(t-4)(t-3)$

$\boxed{t = 4, 3}$ seconds

15. If $= \frac{2x+3}{3x+2}$, then $\boxed{\frac{dy}{dx} = \frac{-5}{(3x+2)^2}}$

$\frac{u'v - uv'}{v^2} = \frac{2(3x+2) - (2x+3)(3)}{(3x+2)^2} = \frac{6x+4-6x-9}{(3x+2)^2}$

16. A particle moves along a line so that its position, in meters, at any time $t \geq 0$, in seconds, is given by

$s(t) = 2t^3 - 11t^2 + 12t - 13$. Show your work and give answers with correct units.

(a) Write the velocity of the particle as a function of time, t. $v(t) = 6t^2 - 22t + 12$

(b) Write the acceleration of the particle as a function of time, t. $a(t) = 12t - 22$

(c) When is the particle at rest? What is its acceleration at these times? $6t^2 - 22t + 12 = 0$

$2(3t^2 - 11t + 6) = 0$

$2(3t-2)(t-3) = 0$

$\boxed{t = 2/3, 3 \text{ seconds}}$

$\boxed{a(2/3) = -16 \quad a(3) = 14}$

(d) When does the particle change direction? Justify your answer.

$(0, 2/3)$ | $(2/3, 3)$ | $(3, \infty)$

$f'(1/3) = 5/3$ | $f'(1) = -4$ | $f'(4) = 20$

move right | move left | moves right

$\boxed{t = \frac{2}{3}, 3 \text{ seconds}}$

17. The graph of y = -5/(x-2) is concave downward for which values of x?

$f(x) = -5(x-2)^{-1}$

$f'(x) = 5(x-2)^{-2}$

$f''(x) = -10(x-2)^{-3}$

$f''(x) = \frac{-10}{(x-2)^3} = 0$ $x = 2$

$(-\infty, 2)$ | $(2, \infty)$

$f''(1) = 10$ | $f''(3) = -10$

Concave up | Concave down

$\boxed{(2, \infty)}$

18. The function defined by $f(x) = x^3 - 3x^2$ for all real numbers x has a relative maximum at x = ?

$f'(x) = 3x^2 - 6x$ $3x(x-2) = 0$

$f''(x) = 6x - 6$ $x = 0, 2$ critical pts

$x = 0$ relative max

$f''(0) = -6$ concave down

$x = 2$ relative min

$f''(2) = 6$ concave up

19. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when x = ?

$0 = x(x+1)(x-2)^2$

$x = 0, -1, 2$ candidates

$(-\infty, -1)$ | $(-1, 0)$ | $(0, 2)$ | $(2, \infty)$

$f''(-2) > 0$ | $f''(-1/2) < 0$ | $f''(1) > 0$ | $f''(3) > 0$

Concave up | Concave down | Concave up | Concave up

$\boxed{x = -1 \text{ and } 0 \text{ poi}}$

change concavity!

20. $\lim_{x\to\infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)} = \frac{-2x^2 + 7x - 3}{1x^2 + 2x - 3}$

-2

21. If $f(x) = \cos(3x)$, then $f'(\pi/9) =$

$f'(x) = -\sin(3x)(3)$

$f'\left(\frac{\pi}{9}\right) = -\sin\left(\frac{3\pi}{9}\right)(3)$

$-3\sin\left(\frac{\pi}{3}\right) \rightarrow -3\left(\frac{\sqrt{3}}{2}\right)$

$\boxed{-\frac{3\sqrt{3}}{2}}$

oops, I forgot the y

22. In the xy-plane, what is the slope of the line tangent to the graph of $x^2 + xy + y^2 = 7$ at (3, 2)?

XXXXX $\boxed{-2}$ Answer should be -8/7

$2x + (1)y + (x)(y') + 2y' = 0$

$2x + y + xy' + 2y' = 0$

$xy' + 2y' = -2x - y$

$y'(x+2) = -2x - y$

$y' = \frac{-2x - y}{x+2}$

$= \frac{-2(3) - (2)}{2+2} = \frac{-8}{4}$

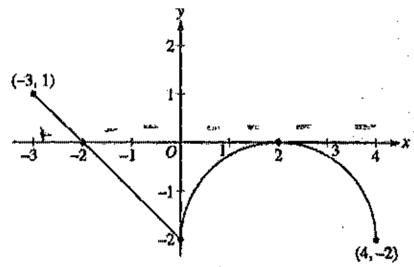
23. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

$v(t) = x'(t) = 2t - 6$

$0 = 2t - 6$

$\boxed{t = 3}$

24. Let f be a function defined on the closed interval $-3 \le x \le 4$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown to the right.



Graph of f'

a) On what intervals, if any, is f increasing?

$(-3, -2)$

b) On what intervals, if any, is f concave upward?

$f''(x) > 0$ $(0, 2)$

c) Find the x–coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$.

$f''(x) = 0$ or DNE $\boxed{x = 0, 2}$ changes concavity!

d) Find an equation for the line tangent to the graph of f at the point (0, 3).

$f'(0) = -2$ $\boxed{y - 3 = -2(x - 0)}$

25. Let f be the function with derivative given by $f'(x) = x^2 - 2/x$. On which of the following intervals is f decreasing?

$\boxed{(0, \sqrt[3]{2})}$

$x \neq 0 \rightarrow$ $x^2 - \frac{2}{x} = 0 \rightarrow x^2 = \frac{2}{x}$

$x^3 = 2$

$x = \sqrt[3]{2}$

$(-\infty, 0)$ | $(0, \sqrt[3]{2})$ | $(\sqrt[3]{2}, \infty)$

$f'(-1) > 0$ | $f'(1) < 0$ | $f'(2) > 0$

increasing | decreasing | increasing

26. If $y = 3x - 6$, what is the minimum value of the product xy?

$P = xy$ rewrite in one term $P = x(3x - 6)$

minimize

$P = 3x^2 - 6x$ $0 = 6x - 6$ $x = 1$ $f''(1) > 0$ min

~~27. Let g be a twice-differentiable function with g'(x) > 0 and g''(x) > 0 for all real numbers x, such that~~ XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

g(4) = 12 and g(5) = 18. Of the following, which is a possible value for g(6) ?

means increasing means concave up

28. If $f(x) = x^2 + 2x$, then $\frac{dy}{dx}(f(\ln x)) =$ $(\ln x)^2 + 2\ln x$

$2\ln x \cdot \frac{1}{x} + \frac{2}{x} = \frac{2\ln x}{x} + \frac{2}{x} = \boxed{\frac{2\ln x + 2}{x}}$

29. If $\cos(xy) = x$, then $dy/dx =$

$-\sin(xy)\,[(1)(y) + (x)(y')] = 1$

$-y\sin(xy) - x\sin(xy)y' = 1$

$\frac{-x\sin(xy)y'}{-x\sin(xy)} = \frac{1 + y\sin(xy)}{-x\sin(xy)}$

$\boxed{y' = \frac{1 + y\sin(xy)}{-x\sin(xy)}}$

30. The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?

$SA = 2\pi r^2 + 2\pi r h$ $\xrightarrow{\text{rewrite}}$ $SA = 2\pi r^2 + 2\pi r\left(\frac{16}{r^2}\right)$

minimize

$V = \pi r^2 h$

$16\pi = \pi r^2 h$ $h = \frac{16}{r^2}$

$SA = 2\pi r^2 + 32\pi r^{-1}$

$0 = 4\pi r - 32\pi r^{-2}$

$r \neq 0 \longrightarrow 4\pi r = \frac{32\pi}{r^2}$

$\frac{4\pi r^3}{4\pi} = \frac{32\pi}{4\pi}$

$r^3 = 8$

$r = 2$

$h = \frac{16}{2^2}$

$h = 4$ in

31. The volume V of a cone ($V = 1/3\,\pi r^2 h$) is increasing at the rate of 28π cubic ft. per second. At the instant when the radius, r, of the cone is 3 ft., its volume is 12π cubic ft. and the radius is increasing at 1/2 ft. per second.

(a) What is the rate of change of the area of its base? (b) What is the rate of change of its height, h?

given

$\frac{dV}{dt} = 28\pi \quad r = 3 \quad V = 12\pi \quad 12\pi = \frac{1}{3}\pi(3)^2 h \quad 4 = h \quad \frac{dr}{dt} = \frac{1}{2} \quad \frac{dA}{dt} = ? \quad \frac{dh}{dt} = ?$

(a)

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(3)\left(\frac{1}{2}\right)$

$\boxed{\frac{dA}{dt} = 3\pi \text{ ft}^2/\text{sec}}$

(b) $V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \frac{1}{3}\pi\left[\left(2r\frac{dr}{dt}\right)(h) + (r^2)\left(\frac{dh}{dt}\right)\right]$

$28\pi = \frac{1}{3}\pi\left[2(3)\left(\frac{1}{2}\right)(4) + (3^2)\frac{dh}{dt}\right]$

$84 = 12 + 9\frac{dh}{dt}$

$72 = 9\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = 8 \text{ ft/sec}}$

32. If $y = 5^x + 4x - 2$, Find dy/dx.

$\frac{dy}{dx} = 5^x \ln 5 + 4$

33. If $f(x) = \ln(x + 2 + e^x)$, then $f'(0)$ is

$f'(x) = \frac{1}{x+2+e^x}(1+e^x)$

$f'(0) = \frac{1+e^0}{2+e^0}$

$\boxed{f'(0) = \frac{2}{3}}$

34. If $y = \cos(2x)$, find dy/dx.

$\frac{dy}{dx} = -\sin(2x)\cdot 2$

35. If $y = x^3 \sin(5x)$, find dy/dx.

$\frac{dy}{dx} = (3x^2)(\sin(5x)) + (x^3)(\cos(5x)\cdot 5)$

36. What is the slope of the line tangent to the curve $2y^2 - x^2 = 3 - 3xy$ at the point (3, 2)?

$\boxed{0}$

$4yy' - 2x = -3[(1)(y) + (x)(y')]$

$4yy' - 2x = -3y - 3xy'$

$+2x \qquad + 3xy'$

$4yy' + 3xy' = -3y + 2x$

$y'(4y + 3x) = -3y + 2x$

$\frac{y'(4y+3x)}{4y+3x} = \frac{-3y+2x}{4y+3x}$

$y' = \frac{-3y+2x}{4y+3x}$

$= \frac{-3(2)+2(3)}{4(2)+3(3)}$

$= \frac{0}{14} = 0$

37. If $f(x) = (\ln x)^2$, then $f''(\sqrt{e}) =$

$f'(x) = 2(\ln x)^1 \cdot \frac{1}{x} = \frac{2}{x}\cdot \ln x$ or $2x^{-1}\ln x$

$f''(x) = (-2x^{-2})(\ln x) + \left(\frac{2}{x}\right)\left(\frac{1}{x}\right)$

38. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point $x = \frac{1}{4}$?

$y' = \frac{1}{1+(4x)^2}\cdot 4$

$f'(1/4) = \frac{4}{1+(4\cdot\frac{1}{4})^2} = \frac{4}{2} = \boxed{2}$

37. $f''(\sqrt{e}) = \left(\frac{-2}{(\sqrt{e})^2}\right)(\ln\sqrt{e}) + \frac{2}{\sqrt{e}}\cdot\frac{1}{\sqrt{e}}$

$= \frac{-\cancel{2}}{e}\cdot\frac{1}{\cancel{2}}\ln e + \frac{2}{e}$

$= -\frac{1}{e} + \frac{2}{e} = \boxed{\frac{1}{e}}$

35. $\frac{dy}{dx} = 3x^2\sin(5x) + 5x^3\cos(5x)$