UNIT 7 & 8 Basic Integration

NAME:________________________

REVIEW

DATE:_______

Unit 7: Approximation Methods

Riemann Sums = Estimation of area under the curve. You need to be able to do left, right, and midpoint using rectangles, usually involves a table.

Trapezoidal Approximation = same as Riemann's but use trapezoids

MULTIPLE CHOICE

1. The graph shows which of the following?

(A) Left hand Riemann Sum with 5 subintervals

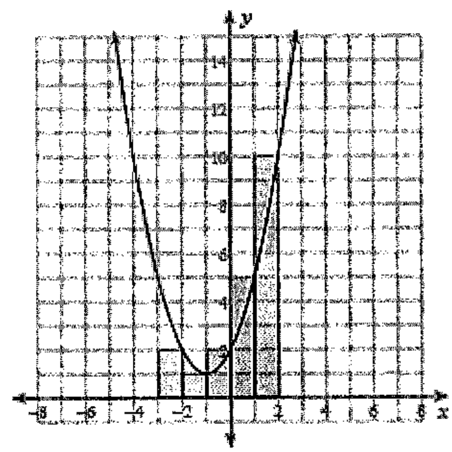
(B) Right hand Riemann Sum with 5 subintervals

B

(C) Midpoint Riemann Sum with 5 subintervals

(D) Trapezoidal Approximation with 5 subintervals

(E) None of the above



FREE RESPONSE

2. Use a left-hand Riemann sum with 4 subintervals to approximate the integral based of the values in the table.

$$\int_0^{10} f(x)dx = 35$$

x	0	4	6	7	10
$f(x)$	3	2	4	5	7

$$4(3) + 2(2) + 1(4) + 3(5)$$

$$35$$

Unit 8: Integration

Integrals are the area under the curve

Indefinite Integrals = are evaluated using antidifferentiation, don't forget C, you can find C if they give you a point on the original curve. $\int f(x)dx$

Definite Integrals = are evaluated using the Fundamental Theorem of Calculus, geometry, or the calculator.

$$\int_a^b f(x)dx$$

When looking for a total area, use absolute values!

Review the properties of definite integrals.

Evaluate the indefinite integrals.

3. $\int \left(\frac{x^3}{4} + \sqrt{x}\right) dx$

$\int \left(\frac{1}{4}x^3 + x^{1/2}\right)dx$

$\frac{1}{16}x^4 + 2x^{3/2} + C$

$\frac{1}{16}x^4 + 2\sqrt{x^3} + C$

4. $\int 3x^{-1}dx = \int \frac{3}{x}\,dx$

$3\ln x + c$

5. $\int (e^x - \sin x)dx$

$e^x + \cos x + c$

Evaluate the definite integrals using Fundamental Theorm of Calculus.

6. $\int_{-4}^{-2} \left(-\frac{x^2}{2} - 3x - \frac{7}{2}\right) dx \quad -\frac{1}{6}x^3 - \frac{3}{2}x^2 - \frac{7}{2}x \Big]_{-4}^{-2}$

$\left[-\frac{1}{6}(-2)^3 - \frac{3}{2}(-2)^2 - \frac{7}{2}(-2)\right] - \left[-\frac{1}{6}(-4)^3 - \frac{3}{2}(-4)^2 - \frac{7}{2}(-4)\right]$

$\left(\frac{8}{6} - 6 + 7\right) - \left(\frac{64}{6} - 24 + 14\right)$

$\left(\frac{8}{6} + 1\right) - \left(\frac{64}{6} - 10\right)$

$\frac{14}{6} - \frac{4}{6} = \frac{10}{6} = \frac{5}{3}$

7. $\int_1^3 \left(\frac{x^3 - x}{3x}\right) dx = \int_1^3 \left(\frac{x^3}{3x} - \frac{x}{3x}\right)dx = \int_1^3 \left(\frac{x^2}{3} - \frac{1}{3}\right)dx$

$= \frac{1}{9}x^3 - \frac{1}{3}x \Big]_1^3$

$\left[\frac{1}{9}(3)^3 - \frac{1}{3}(3)\right] - \left[\frac{1}{9}(1)^3 - \frac{1}{3}(1)\right]$

$(3 - 1) - \left(\frac{1}{9} - \frac{1}{3}\right)$

$2 + \frac{2}{9} = \frac{20}{9}$

Answer the following.

8. Given that $f'(x) = \frac{1}{2}x^2 + \frac{3}{4}x$ and $f(1) = 2$. Find $f(x)$.

$f(x) = \frac{1}{6}x^3 + \frac{3}{8}x^2 + c$

$2 = \frac{1}{6}(1)^3 + \frac{3}{8}(1)^2 + c$

$2 = \frac{1}{6} + \frac{3}{8} + c$

$2 = \frac{13}{24} + c$

$-\frac{13}{24} \quad -\frac{13}{24}$

$c = 35/24$

$f(x) = \frac{1}{6}x^3 + \frac{3}{8}x^2 + \frac{35}{34}$

9. A particle moves along a coordinate line. Its accelartion function is $a(t) = 6t - 22$ for $t \geq 0$. If $v(0) = 24$ find the velocity at $t = 4$.

$v(t) = 3t^2 - 22t + c$

$24 = 3(0)^2 - 22(0) + c$

$24 = c$

$v(t) = 3t^2 - 22t + 24$

$v(4) = 3(4)^2 - 22(4) + 24$

$v(4) = -16$

TEST PREP
NO CALCULATOR

1. $\int \frac{1}{x^2} dx =$ $\quad \int x^{-2} dx = -x^{-1} + C$

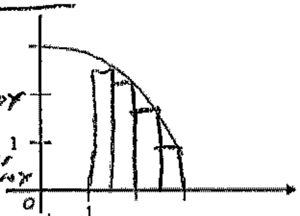
D

(A) $\ln x^2 + C$
(B) $-\ln x^2 + C$
(C) $x^{-1} + C$
(D) $-x^{-1} + C$
(E) $-2x^{-3} + C$

2. The graph of function f is shown below for $0 \le x \le 3$. Of the following, which has the least value?

C

(A) $\int_1^3 f(x)dx$ area under the curve from 1 to 3
(B) Left Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length over approx
(C) Right Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length under approx
(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length good
(E) Trapezoidal sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length good



Graph of f

3. $\displaystyle\int_0^{\frac{\pi}{4}} \sin x \, dx$ $= -\cos x \Big]_0^{\pi/4} = \left(-\cos\left(\frac{\pi}{4}\right)\right) - \left(-\cos(0)\right)$

$\left(-\frac{\sqrt{2}}{2}\right) - (-1)$

D

(A) $-\frac{\sqrt{2}}{2}$
(B) $\frac{\sqrt{2}}{2}$
(C) $-\frac{\sqrt{2}}{2} - 1$
(D) $-\frac{\sqrt{2}}{2} + 1$
(E) $\frac{\sqrt{2}}{2} + 1$

CALCULATOR ACTIVE

4. If $\int_{-5}^{2} f(x)dx = -17$ and $\int_5^2 f(x)dx = 4$, what is the value of $\int_{-5}^{5} f(x)dx$?

A

(A) -21
(B) -13
(C) 0
(D) 13
(E) 21

$\int_2^5 f(x)\,dx = -4$

$\int_{-5}^{2} f(x)\,dx + \int_2^5 f(x)\,dx = \int_{-5}^{5} f(x)\,dx$

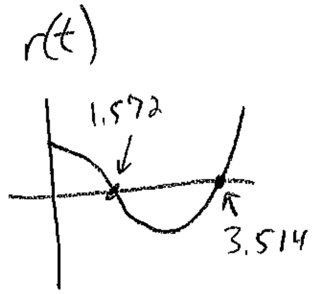
$-17 + -4 =$

-21

CALCULATOR ACTIVE

5. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

r(t)



decreasing from 1.572 to 3.514

(A) $\int_{1.572}^{3.514} r(t)dt$

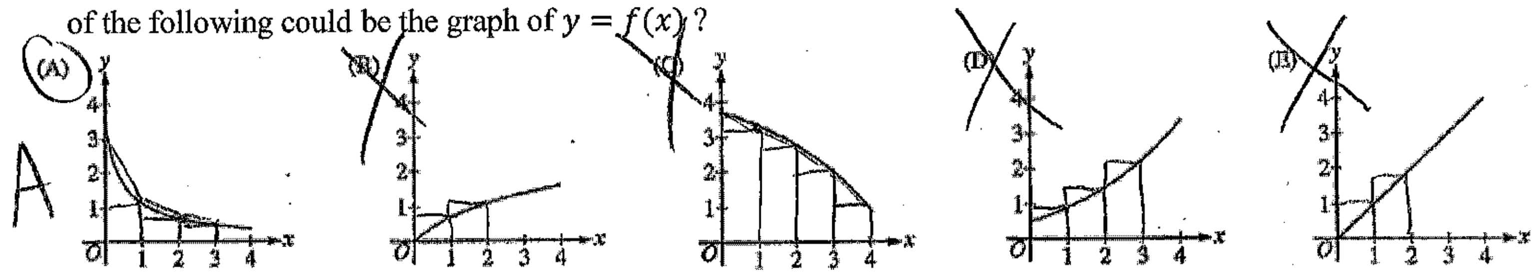
(B) $\int_0^8 r(t)dt$ 0

A (C) $\int_0^{2.667} r(t)dt$

(D) $\int_{1.572}^{3.514} r'(t)dt$

(E) $\int_0^{2.667} r'(t)dt$

6. If a trapezoidal sum overapproximates $\int_0^4 f(x)dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)dx$, which of the following could be the graph of $y = f(x)$?



A

FREE RESPONSE
CALCULATOR ACTIVE

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsisus)	66	60	52	44	43

7. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measure in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) ~~Using correct units, explain the meaning of $\int_0^{10} H(t)dt$ in the context of this problem.~~ Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\int_0^{10} H(t)dt$.

$$\left[2\left(\frac{66+60}{2}\right) + 3\left(\frac{60+52}{2}\right) + 4\left(\frac{52+44}{2}\right) + 1\left(\frac{44+43}{2}\right)\right]$$

$$126 + 168 + 192 + 43.5 = 529.5$$

(b) Using correct units, explain the meaning of $H'(7)$. Use the table to approximate $H'(7)$. Show your calculations.

Change in temperature °C over minutes at 7 minutes

$$\frac{f(9) - f(5)}{9-5} = \frac{44-52}{9-5} = \frac{-8}{4} = -2\,°C/min$$